

On the Dynamics of Atmospheric Bores

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ABSTRACT: The dynamics of a prototypical atmospheric bore are investigated through a series of two-dimensional numerical simulations and linear theory. These simulations demonstrate that the bore dynamics are inherently finite amplitude. Although the environment supports linear trapped waves, the supported waves propagate in roughly the opposite direction to that of the bore. Qualitative analysis of the Scorer parameter can therefore give misleading indications of the potential for wave trapping, and linear internal gravity wave dynamics do not govern the behavior of the bore. The presence of a layer of enhanced static stability below a deep layer of lower stability, as would be created by a nocturnal inversion, was not necessary for the development of a bore. The key environmental factor allowing bore propagation was the presence of a low-level jet directed opposite to the movement of the bore. Significant turbulence developed in the layer between the jet maximum and the surface, which reduced the low-level static stability behind the bore. Given the essential role of jets and thereby strong environmental wind shear, and given that idealized bores may persist in environments in which the static stability is constant with height, shallow-water dynamics do not appear to be quantitatively applicable to atmospheric bores propagating against low-level jets, although there are qualitative analogies.

KEYWORDS: Gravity waves; Mesoscale processes; Nonlinear dynamics; Solitary waves; Turbulence; Waves, atmospheric

1. Introduction

Atmospheric bores are disturbances whose passage is accompanied by a pressure rise and a semipermanent upward displacement of the isentropic surfaces (Knupp 2006; Haghi et al. 2017). A series of waves often trails behind the bore's leading edge, and in contrast to density currents, the near-surface temperature remains relatively unchanged, or even warms, after the bore passes. One of the most spectacular and well-studied examples of an atmospheric bore is the "morning glory," which occurs early in the morning hours in the Gulf of Carpentaria region of northeastern Australia in austral spring (Clarke 1972; Clarke et al. 1981; Smith 1988). Atmospheric bores also occur frequently in the nocturnal environment over the Great Plains of the United States, where they are often initiated by gust fronts and density currents in thunderstorm outflows (Weckwerth et al. 2004; Wilson and Roberts 2006; Koch et al. 2008a,b; Haghi et al. 2017). In favorable conditions, these nocturnal bores can propagate hundreds of kilometers and trigger new convection through low-level lifting that can grow upscale into large organized convective systems (Karyampudi et al. 1995; Koch and Clark 1999).

Because of their important role in initiation of nocturnal convection, observations of atmospheric bores were collected during two field campaigns in the central United States: the International H₂O Project (IHOP) in 2002 (Weckwerth et al. 2004) and Plains Elevated Convection at Night (PECAN) in 2015 (Geerts et al. 2016; Haghi et al. 2019). The ubiquitous nature of nocturnal bores was documented in the data collected during IHOP. Haghi et al. (2017) found that among all radar-detected fine lines, 90% of the convectively generated density currents that could be confidently classified generated a bore that propagated out ahead of the

density current itself. An alternative way to appreciate the frequency of bore generation by convective outflows during IHOP is to compare the pair of spatial histograms in Fig. 1. The density of thunderstorms with radar echoes exceeding 40 dBZ in which the convective outflow did trigger a bore (Fig. 1a) substantially exceeds the number that did not (Fig. 1b). We will be guided by observations from IHOP in our formulation of a prototypical bore environment.

Theories for the dynamics of atmospheric bores have largely relied on analogies with shallow-water flow or on linear theory for internal gravity waves [Haghi et al. (2017) provides a survey]. Most quantitative applications of shallow-water theory to the atmosphere replace the full gravitational restoring force g acting on displacements of an air-water interface with a "reduced gravity" g' acting on displacements of an elevated inversion. Letting $\Delta\theta$ be the jump in potential temperature across the inversion, the reduced gravity is defined as $g' = g\Delta\theta/\theta_0$, where θ_0 is a constant, representative of the mean potential temperature near the interface. The fidelity with which reduced-gravity shallow-water (RGSW) theory can represent actual atmospheric flows depends on the strength and sharpness of the inversion and the extent to which restoring forces generated by the displacement of the inversion dominate those produced by vertical displacements in the continuously stratified layers above and below that inversion (Durrán 2000; Jiang 2014). The contrast between these cases is illustrated by the schematic temperature profiles in Fig. 2. In the case of an elevated inversion sandwiched between two neutrally stratified layers (Fig. 2a), buoyancy restoring forces are generated entirely by the displacement of the elevated inversion, but when the layers above and below the inversion are also stably stratified (Figs. 2a,b), buoyancy restoring forces are also generated by the internal wave displacements within those layers, and such additional buoyancy forces are not included in RGSW theory.

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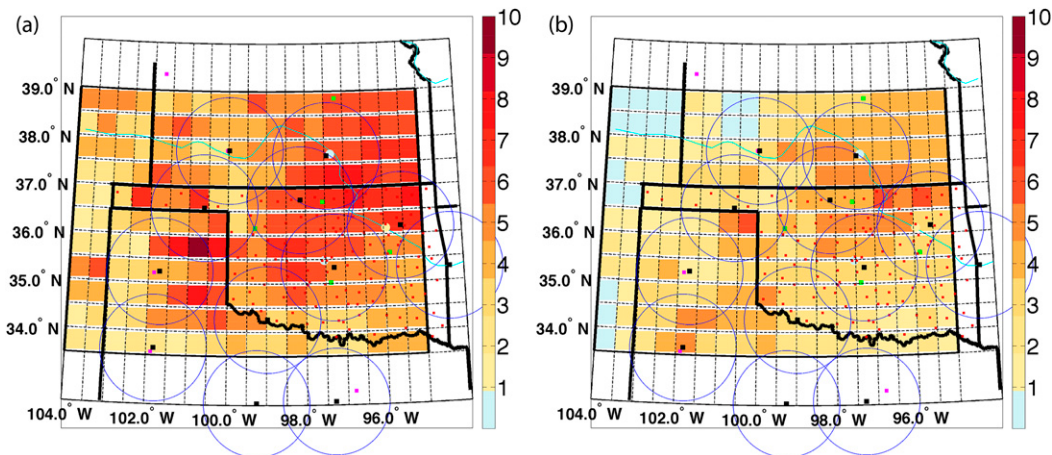


FIG. 1. Spatial histogram of nocturnal convection over the IHOP_2002 field campaign domain. Convection was counted for dBZ > 40 and if the convection generated a convective outflow that (a) developed a bore or (b) did not develop a bore. Circles of radius approximately 120 km depict radar volumes from which the reflectivity was obtained and the red, green, and magenta points indicate observation sites used to diagnose the nature of the observed fine lines in reflectivity.

Leaving aside the problem of stable stratification above the inversion, the environments in which the morning glory and nocturnal Great Plains bores occur typically have surface-based inversions (Fig. 2c), rather than a sharp thin elevated inversions capping a layer of low static stability, as would be most appropriate for the direct application of RGSW theory. RGSW theory has been therefore applied with $\Delta\theta$ representing the change θ across the full depth of the layer (Rottman and Simpson 1989). In this situation, in which the interface is smeared over a layer 0.5–2 km deep, the RGSW model can fail quantitatively, but still provide a useful qualitative analog (Durrán 1986). At least within this qualitative context, enhanced low-level static stability is widely regarded as a key element in atmospheric bore dynamics. We will test the importance of low-level stable layers in section 4.

Theoretical models of linear internal gravity waves easily accommodate multiple layers with continuous density stratification as well as complex vertical profiles of the environmental wind parallel to the direction of bore propagation. The

application of linear-wave theory to atmospheric bores typically focuses on whether conditions are favorable for trapping wave energy near the surface (Crook 1986, 1988). One weakness of this approach is the small-amplitude assumption, which is not satisfied for many observed bores. A second concern is that most previous studies have relied on qualitative assessments of the suitability of the environment for wave trapping (Koch et al. 1991; Martin and Johnson 2008; Koch et al. 2008a,b; Toms et al. 2017; Blake et al. 2017), instead of solving the eigenvalue–eigenfunction problem to rigorously determine whether the environment actually supports trapped waves. We will test the relevance of linear internal gravity wave analysis in sections 3 and 5.

Observations, numerical simulations, and laboratory experiments (Rottman and Simpson 1989; Knupp 2006; Koch et al. 2008a; Johnson and Wang 2019) have shown that bores can be accompanied by turbulent mixing. Such mixing is required to produce the modest increases in surface temperatures and changes in surface dewpoints that are sometimes observed in nocturnal bores. Turbulent mixing also helps

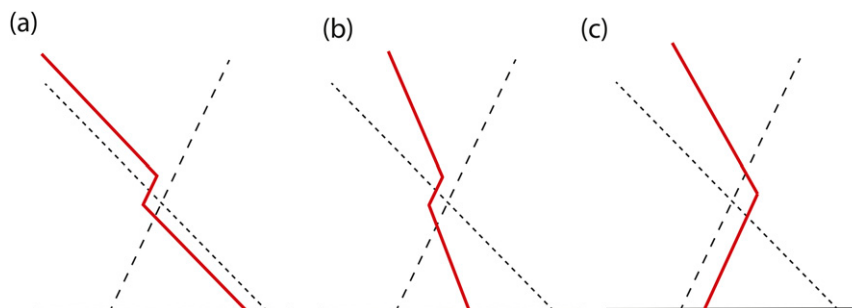


FIG. 2. Schematic skew T diagrams illustrating (a) an elevated isothermal inversion between two neutral layers, (b) an elevated isothermal inversion between two stably stratified layers, and (c) a surface-based isothermal inversion beneath a stably stratified layer. The red curves are the temperature profiles; the long-dashed line and short-dashed lines are isotherms and dry adiabats, respectively.

reduce the static stability throughout the layer behind the leading edge of the bore, and is therefore also an important agent for enhancing nocturnal convection in regions penetrated by the bore (Loveless et al. 2019). Turbulent mixing plays an important role in stabilizing the number of decaying oscillations behind the leading edge of the bore in weakly nonlinear models based on viscous generalizations of the Korteweg–de Vries (KDV) or Benjamin–Davis–Ono equations (Christie 1989). In such generalizations, the turbulence is incorporated by a viscosity large enough to represent an eddy viscosity, but typically uniform throughout the fluid in contrast to the distributions of such viscosity within numerical simulations of atmospheric bores (Koch et al. 2008a; Johnson and Wang 2019). In the case of the KDV equation, the addition of viscosity is required to obtain bore-like solutions in which there is a change in fluid depth across the bore (Whitham 1974, 482–484). The nature and importance of turbulent mixing in our simulations will be discussed in section 6.

The remaining sections of the paper are organized as follows. Section 2 provides details of our numerical model. Section 3 contains the trapped-wave analysis. Section 4 presents the simulation of bores triggered by a cold pool in various environments. Section 5 demonstrates the importance of finite amplitude in the maintenance of the bores. Section 6 explores the importance of turbulent mixing, and section 7 contains the conclusions.

2. The numerical model

Idealized numerical simulations were conducted using the nonhydrostatic mesoscale model CM1 version 1.19 (Bryan and Fritsch 2002) in a dry two-dimensional x – z configuration, with grid spacing $\Delta z = 100$ m and $\Delta x = 500$ m for the basic bore simulations. The trapped wave simulations were conducted with $\Delta x = 250$ m to ensure the waves were well resolved. The pressure was computed using the time-split vertically implicit method of Klemp and Wilhelmson (1978) with a large time step of 8 s. To ensure that vertically propagating gravity waves were absorbed and not spuriously amplified or trapped by reflections at the upper boundary, the top 15 km of the 35-km-deep numerical domain was devoted to a damping layer with an inverse e -folding time scale of $1/600$ s^{−1}. The width of the horizontal domain was 1440 km. Subgrid-scale turbulence was parameterized by integrating an equation for the evolution of subgrid-scale turbulent kinetic energy (TKE) to determine horizontal and vertical eddy diffusion coefficients. No PBL scheme was employed, consistent with Johnson and Wang (2019), who obtained superior bore simulations when near-surface mixing was determined by parameterized eddy diffusion instead of a PBL scheme. The surface boundary conditions were free slip (the velocity component normal to the surface was set to zero), and there were no surface fluxes of heat or momentum.

3. Trapped wave analysis

Under the Boussinesq approximation, vertical velocities of the form

$$w' = \hat{w}(z)e^{ik(x-ct)}, \quad (1)$$

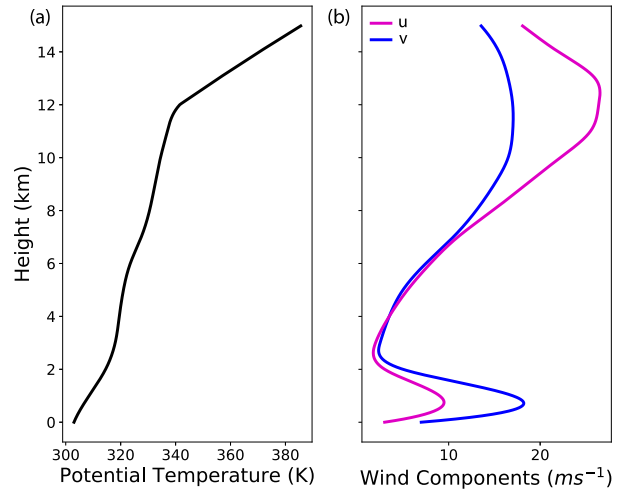


FIG. 3. TW-composite sounding obtained by smoothing five soundings collected around 0900 UTC 4 Jun 2002 from the IHOP field experiment network: (a) potential temperature and (b) u - and v -wind components.

where k is the wavenumber and c is the phase speed, will satisfy the linearized Euler equations for two-dimensional stratified (x – z) flow, with a basic-state horizontal wind speed $U(z)$, if $\hat{w}(z)$ satisfies the Taylor–Goldstein equation

$$\hat{w}_{zz} + (l_s^2 - k^2)\hat{w} = 0, \quad (2)$$

where the subscript z denotes a derivative with respect to the vertical coordinate, and

$$l_s^2 = \frac{N^2}{(U - c)^2} - \frac{U_{zz}}{U - c} \quad (3)$$

is the square of the Scorer parameter generalized for waves with nonzero phase speed. Here $N(z)$ is the Brunt–Väisälä frequency defined such that $N^2 = g\bar{\theta}_z/\bar{\theta}_0$, with g the gravitational acceleration, $\bar{\theta}$ the environmental profile of potential temperature, and $\bar{\theta}_0$ a constant reference potential temperature.

Linear internal gravity waves can be trapped near the surface if there is a sufficient decrease in l_s with height. Many previous investigations of the potential for wave trapping in a given prebore environment (e.g., Crook 1988) provide heuristic discussions of the Scorer-parameter profile, noting for example that higher static stability near the surface, and a low-level jet directed opposite to the movement of the bore, would both tend to produce larger values of l_s near the surface, and thereby help trap the wave. The extent to which linear waves are *actually* trapped in any given environment can be determined by solving the eigenvalue-eigenfunction problem obtained from (2) together with the boundary condition $\hat{w} = 0$ at $z = 0$ and an upper boundary condition specifying either exponential decay or upward energy propagation as $z \rightarrow \infty$, depending on the sign of $l_s^2 - k^2$.

We solved this eigenvalue-eigenfunction problem using the solver in Durran et al. (2015) for the trapped standing mountain-wave case (in which $c = 0$) after generalizing it to allow the specification of arbitrary values of c . The potential for trapped

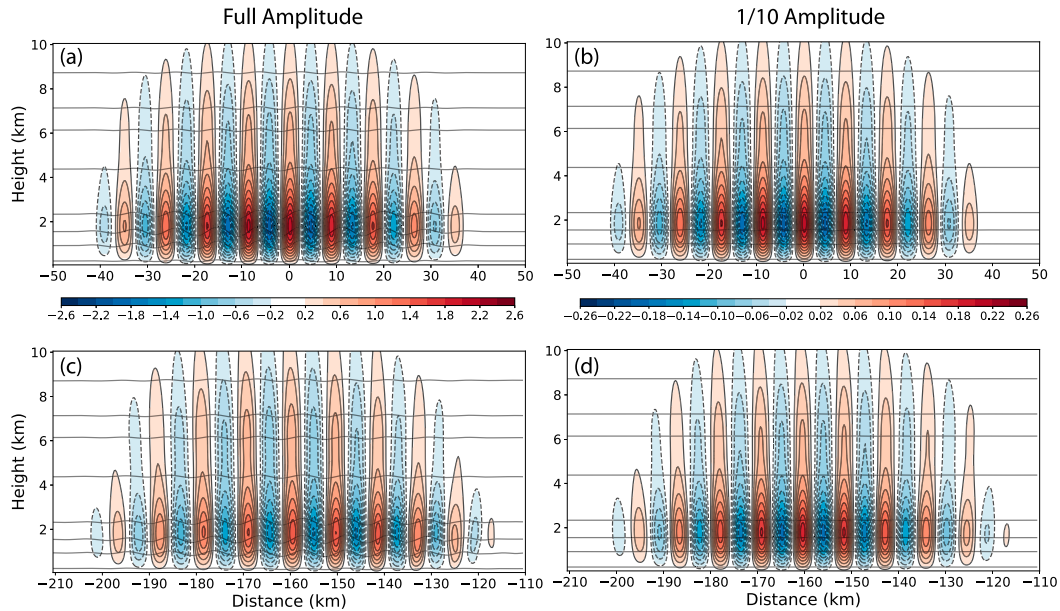


FIG. 4. Trapped wave packet simulations for the TW-composite sounding with environmental winds projected onto the line running from 310° to 130° (positive toward the southeast). Vertical velocity (color contours; in m s^{-1}) and isentropes (contoured every 4 K beginning at 304 K). (left) Full-amplitude simulation, at hours (a) 0 and (c) 6. (right) Simulation with all perturbation amplitudes reduced a factor of 0.1, at hours (b) 0 and (d) 6. In the right column, the velocity contours are reduced by a factor of 0.1 compared to the left column.

waves to propagate in all directions was evaluated for a vertically smoothed composite of five special soundings collected around 0900 UTC 4 June 2002 from the IHOP field experiment network, at which time a bore was observed moving through the network from the north-northwest (Koch et al. 2008a; Fig. 3). This composite sounding, shown in Fig. 3 and hereafter denoted as the trapped-wave composite (TW composite), has enhanced static stability throughout the lowest 2 km, where the Brunt–Väisälä frequency N is roughly 0.014 s^{-1} . Between 2 and 12 km, N has a typical tropospheric value of about 0.009 s^{-1} , above which the lower stratosphere is nearly isothermal ($N = 0.02 \text{ s}^{-1}$). A low-level jet is blowing out of the south-southwest with peak wind speeds at about 600 m above ground level (AGL). Smoothing the TW-composite sounding ensures that the Scorer parameter is not dominated by finescale, yet relatively low-amplitude fluctuations in $U(z)$. These low-amplitude finescale fluctuations can dominate the value of I_s^2 through the U_{zz} term but are unlikely to represent perturbations that are horizontally coherent over large horizontal scales, as should be the case for features included in the basic state about which the governing equations are linearized.

Searching for trapped waves propagating in all azimuthal directions and over a broad range of phase speeds,¹ the only directions toward which such waves could be propagating were found in a wedge centered around 310° true, i.e., moving toward

the northwest, which is almost opposite the direction of the observed bores. There are no linear trapped wave solutions propagating in the direction of the bore. Letting the wavenumber k be the eigenvalue obtained from the solver, resonant wavelengths $2\pi/k$ for these waves were roughly 9 km and their phase speeds were about 15 m s^{-1} . Numerical simulations of w and θ in two representative wave packets propagating toward 310° with a phase speed of 15 m s^{-1} and wavelength $2\pi/k = 8.8 \text{ km}$ are shown in Fig. 4. The w field was initialized using the vertical eigenfunction structure from the solver after multiplying by $[\rho_0/\bar{p}(z)]^{1/2}$ to roughly compensate for the Boussinesq approximation used in the theoretical analysis. The maximum w was 2 m s^{-1} in the packet shown in the left column and 0.2 m s^{-1} in that shown on the right. The initial fields for the perturbation u , θ and pressure were computed from w using the polarization relations for internal gravity waves. The periodic wave train of the form (1) was multiplied by the envelope function $\cos(kx/10)$ to create a series of localized wave packets with 10 waves in each packet, as shown in a 100-km-wide subdomain centered at $x = 0$ in Figs. 4a and 4c.

The evolution of these wave packets was simulated numerically using CM1 as described previously. The horizontal axis in Fig. 4 runs toward the southeast (from 310° to 130°), in which case the environmental winds are negative, and the phase speed c is -15 m s^{-1} . The packet propagates at the group velocity, which was estimated as -7.5 m s^{-1} by solving the eigenvalue problem with two slightly different specified values of c and evaluating the finite-difference approximation

¹ Phase speeds and azimuthal directions for which a critical layer would be present were omitted from the search.

$$\frac{\Delta\omega}{\Delta k} = \frac{\Delta(ck)}{\Delta k} = \frac{\Delta(c)}{\Delta k}k + c. \quad (4)$$

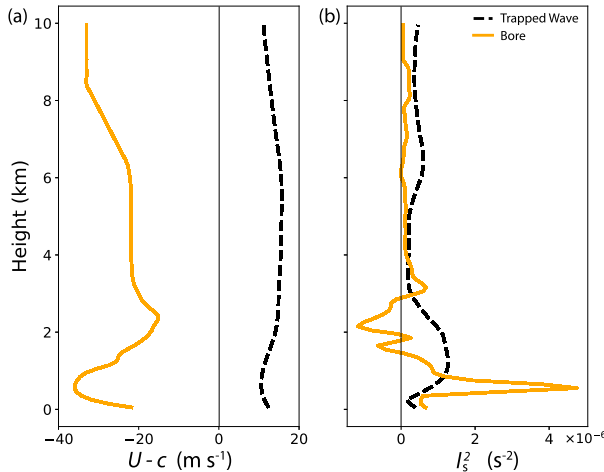


FIG. 5. Vertical profiles of (a) relative wind and (b) Scorer parameter squared for the trapped wave (dashed black) and the bore (solid orange) environments.

Figures 4b and 4d show the simulated evolution of the lower-amplitude wave packet over a period of 6 h, during which time the packet propagates 161 km to the northwest, implying an average propagation speed closely matching the computed group velocity of -7.5 m s^{-1} . The packet undergoes only slight changes in its amplitude and structure, with the strength of the maximum updraft decreasing by about 10% from the initial value of 0.2 m s^{-1} . The slight unsteadiness in the low-amplitude simulation is likely due to a combination of numerical error, weak finite-amplitude effects, and errors from specifying initial conditions using the Boussinesq approximation without perfectly compensating for compressibility. To slightly exceed the 1.5 m s^{-1} vertical velocity amplitude (and therefore the non-linearity) of the bore observed at the time of the composite soundings (Koch et al. 2008a; Fig. 4b), Figs. 4a and 4c show the corresponding 6-h changes for a packet with an initial maximum w of about 2 m s^{-1} . Once again, the packet approximately maintains its shape while traveling 159 km to the northwest. Nonlinear effects are, nevertheless, more pronounced in this case as the maximum w decreases by about 30% over the 6 h period.

The relative wind $U - c$ for the trapped-wave simulation is shown as the dashed black line in Fig. 5a. The mean winds directed toward 310° are weak, so the biggest contribution to $U - c$ comes from the -15 m s^{-1} phase speed of the wave itself. The square of the Scorer parameter is plotted as a function of height as the dashed black line in Fig. 5b; l_s^2 is substantially larger in the layer $500 \leq z \leq 2500 \text{ m}$ primarily because of the enhanced static stability in the lowest 2.5 km (Fig. 3a). The decrease in the Scorer parameter with height, together with the relatively deep layer of high l_s near the surface, is favorable for wave trapping. The orange lines in Fig. 5 show the corresponding plots for the bore simulation which will be discussed in the next section.

In summary, the results in this section show that both linear and nonlinear trapped waves can be supported in an environment representative of the observed bore, but the range of

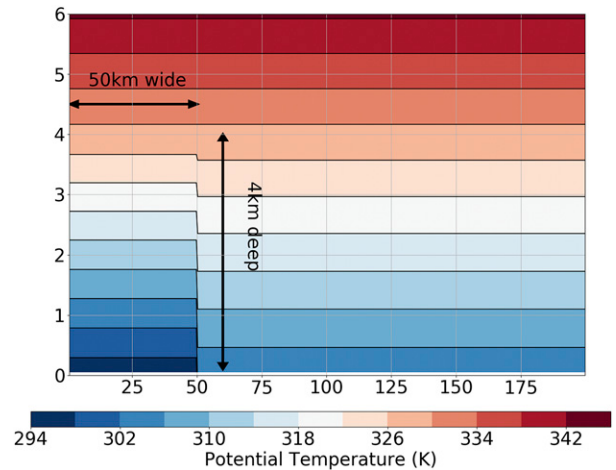


FIG. 6. Schematic of the initial state of CM1 for density current-generated bore simulations. Filled contours are potential temperature. The staggered potential temperature contours indicate the presence of a rectangular blob of colder air relative to its environment that is 50 km wide and 4 km deep. The x axis is horizontal distance (km) and z axis is the vertical distance (km).

directions along which those waves can propagate are limited and almost opposite to the direction of the bore. It follows that the observed bore was not the product of linear trapped-gravity wave dynamics.

4. Bores triggered by a cold pool

a. Initial conditions

In this section, we continue to explore the essential dynamics of atmospheric bores of the Great Plains in a representative prototypical environment and in idealized simplifications of that environment. The bores were triggered artificially by a density current created through the collapse of a 4-km-tall, 50-km-wide block of cold air at the left (northern) edge of the domain. At the initial time, this block was 7.5 K colder than the environment at the surface and the temperature deficit decreased linearly with height to zero at 4 km, as illustrated in Fig. 6. Open lateral boundary conditions effectively extended the cold-air pool into the region $x < 0$.

The horizontally homogeneous environmental conditions outside of the initial block of cold air were taken from a prototypical low-level jet (LLJ) sounding composited from the same five prebore IHOP soundings used to create the trapped-wave composite sounding (Fig. 3). In contrast to the TW composite, the vertical profile of the winds in the LLJ composite is less strongly smoothed² and is designed to capture the structure of the jet with the wind maximum again near 600 m AGL. The LLJ-composite sounding is plotted in Fig. 7; here the winds are the north-south component of the total wind, with negative signs corresponding to winds toward the bore, or

² We did not wish to reduce the magnitude of the jet maximum, and we are not as concerned about the magnitude of U_{zz} .

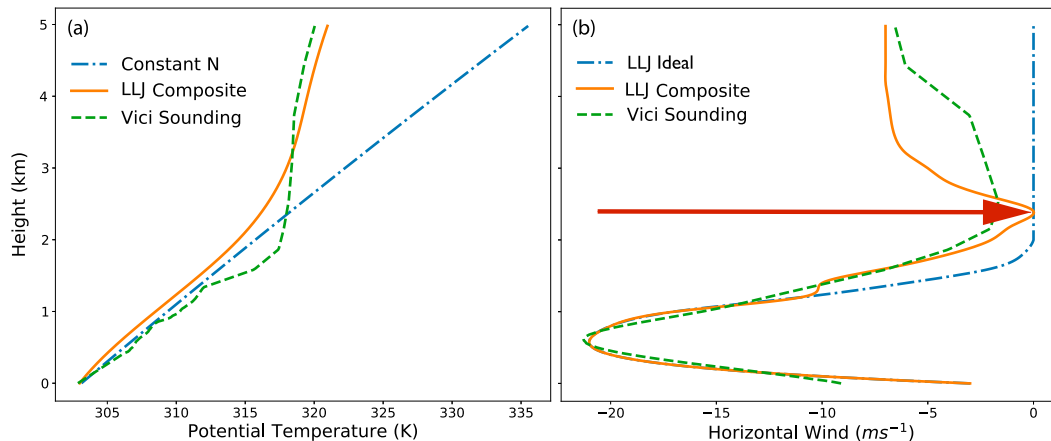


FIG. 7. Environmental soundings for the bore analysis: (a) potential temperature and (b) horizontal wind speed in the direction of bore propagation. LLJ composite (orange solid line), Vici sounding (green dashed line), and idealized sounding (blue dot-dashed line). The heavy red arrow shows the windshear vector between the elevation of the jet maximum and $z = 2.5$ km, which is in the direction of bore propagation and will be discussed in the conclusions (section 7).

equivalently, winds from the south. It may be seen that this composite does capture the essential features of individual soundings by comparing it with the Vici, Oklahoma, sounding, which is also plotted in Fig. 7. One additional idealized thermodynamic sounding is also plotted in Fig. 7a. This idealized profile, for which $N = 0.014 \text{ s}^{-1}$, closely matches the composite static stability in the lowest 2 km and keeps N constant with height to the top of the domain. Finally, an idealized wind profile is also plotted in Fig. 7b that approximates the low-level jet through the lowest 1.8 km and drops to zero above that level.

b. Environmental conditions essential for the bore

Figures 8a and 8b show the bore and cold-pool evolution at hours 2 and 4 in the case with the LLJ-composite sounding representative of the prebore conditions on 4 June 2002, zoomed in to a pair of slightly different subdomains within the region $25 \leq x \leq 275$ km. The cold pool, indicated by the region of purple fill in the lower-left corner of each panel, propagates slowly southward; its leading edge extends to approximately $x = 110$ km after 4 h. The most impressive feature is not, however, the disturbance associated with the leading edge of the cold pool, but rather a bore that has propagated well away from the cold pool. After 4 h, the leading edge of the bore is more than 100 km beyond the leading edge of the cold pool. Inspection of the isentropes and vertical velocity field (color contours) shows that the structure of the leading edge of the bore remains constant between hours 2 and 4. Even at times as late as hour 10, the disturbance structure at the leading edge of the bore remains essentially unchanged (not shown).

What environmental factors are essential for the support of this bore? The updraft at the leading edge of the bore propagates horizontally at about 15 m s^{-1} ; using this value for the phase speed, the relative wind $U - c$ and f_s^2 are plotted for the bore as the orange lines in Fig. 5. Although it is noisier, the overall structure of the f_s^2 profile is roughly similar to the idealized case considered by Crook (1988), with some large values

at and below the jet maximum and a region of negative f_s^2 above the jet. However, as discussed in the previous section, no trapped waves are actually supported by this Scorer parameter profile. Another difference from the linear theory for trapped internal gravity waves is that the phase speed and the group velocity are identical for the updraft at the leading edge of the bore, as would be the case in shallow-water hydraulic theory. There is, however, no elevated inversion in this environment that could be used to compute a reduced gravity and an effective depth for the lower layer. A similar situation occurs in some “morning glory” events (Clarke et al. 1981), and these have been analyzed in Rottman and Simpson (1989) by assuming the analog of the free surface in shallow-water theory occurs where there is a pronounced decrease in N at the top of the surface-based stable layer.

How important is the surface-based stable layer and the abrupt reduction in static stability at $z = 2$ km to the maintenance and propagation of this bore? Figures 8c and 8d show the disturbance that develops if the potential temperature in the environment is left unchanged, while the -22 m s^{-1} low-level jet is replaced with a -10 m s^{-1} uniform-with-height flow. The cold-pool propagation is grossly similar to that in the previous case, although it does move slightly faster and remains somewhat deeper. The bore, on the other hand, has completely disappeared and is replaced by a very broad region of sloping isentropes that gradually rise to meet the oncoming cold pool. Figure 8c suggests the disturbance in front of the cold pool has some resemblance to a vertically propagating internal gravity wave, but it clearly looks nothing like the solution in Figs. 8a and 8b. Evidently the low-level stable layer is not, in isolation, sufficient to support the bore.

The influence of the other distinctive feature in the LLJ-composite sounding, the low-level jet, is isolated in the simulation shown in Figs. 8e and 8f. In this case the static stability is independent of height and specified as $N = 0.014 \text{ s}^{-1}$, which is representative of the average N throughout the layer below $z = 2$

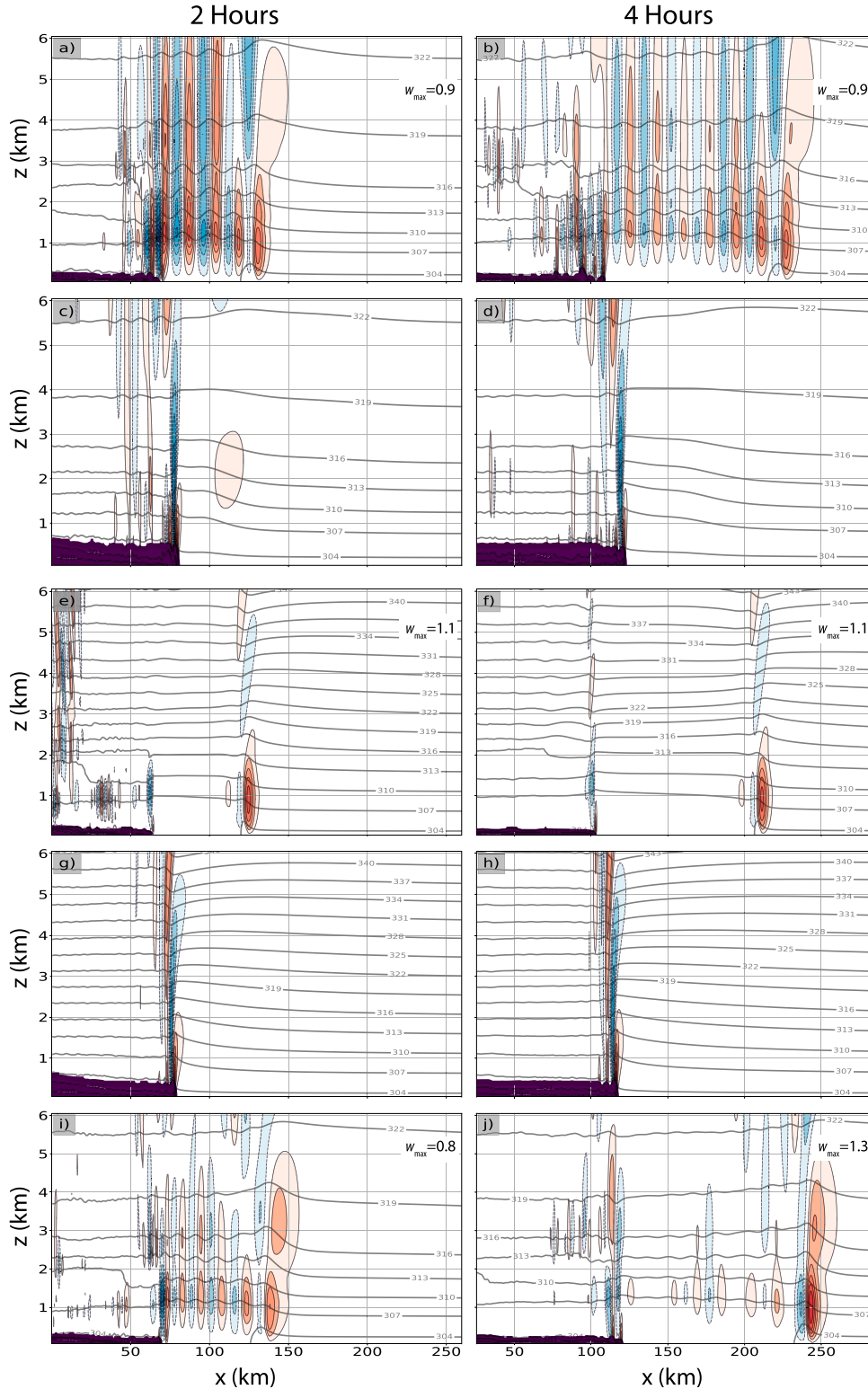


FIG. 8. Disturbances generated by the dam-break initialization for environments with (a),(b) LLJ-composite θ and U ; (c),(d) LLJ-composite θ , $U = 10 \text{ m s}^{-1}$; (e),(f) constant- N , ideal jet; (g),(h) constant- N , $U = 10 \text{ m s}^{-1}$; and (i),(j) LLJ-composite θ , ideal jet. Times are (left) $t = 2 \text{ h}$ and (right) $t = 4 \text{ h}$. Isentropes of potential temperature are plotted every 3 K; purple indicates cold-pool air for which $\theta < 303 \text{ K}$. Color fill shows vertical velocities (red is up, blue is down) contoured at 0.2 m s^{-1} intervals with the zero contour omitted. The maximum vertical velocities (m s^{-1}) at the leading edge of the bore are given in each bore case.

in the LLJ-composite sounding. The wind profile is the ideal jet plotted in Fig. 7b. The propagation of the cold pool in this case is very similar to that for the full LLJ-composite sounding, and once again a constant-amplitude bore develops and propagates well ahead of the cold pool. The 1.1 m s^{-1} maximum upward velocity at the leading edge of the bore is slightly stronger than the 0.9 m s^{-1} maximum that developed LLJ-composite-sounding case. Another notable difference between this case and the simulation for the LLJ-composite sounding is that there are no additional waves behind the bore's leading edge. The additional waves in the LLJ-composite-sounding case are partially trapped by the reduction in static stability aloft, but as apparent in Figs. 8e and 8f, low static stability aloft is not required to support the bore itself.

The importance of the low-level jet for the maintenance of the bore is further demonstrated in a simulation with N once again set to 0.14 s^{-1} and constant with height, while the jet structure is removed and replaced with a vertically uniform -10 m s^{-1} wind speed. As shown in Figs. 8g and 8h, there is no hint of a bore in this simulation. The speed of the cold pool is, nevertheless, very similar to that for the case in Figs. 8c and 8d, which is not surprising because in both cases the low-level wind and static stability are almost identical.

A final case of interest is shown in Figs. 8i and 8j, in which the temperature profile is again from the LLJ-composite sounding (as in Figs. 8a–d) while the wind profile is given by the ideal jet (as in Figs. 8e and 8f). As might be expected, a bore once again develops, but surprisingly, this bore gradually amplifies with time as it propagates away from the cold pool. The reason for this amplification is not entirely clear. In comparison to the bore generated using the full LLJ-composite wind profile, the bore in this simulation is losing less energy through vertical propagation in the *main wave above the leading edge of the bore* (cf. the vertical velocities in the leading updraft/downdraft couplet at the top of the domain in Figs. 8b,j). On the other hand there is certainly faster leakage of energy out of the waves trailing behind the front of the bore. The ideal-jet environment is less suitable for inhibiting the propagation of the trailing waves because the decrease in the bore-relative oncoming winds above $z = 2.5 \text{ km}$ in the ideal profile relative to those in the LLJ-composite sounding (Fig. 6b) increases the Scorer parameter in the upper layer. A second reason that the atmosphere is less favorable for partially trapping the trailing waves in the ideal-jet environment is that it lacks the curvature U_{zz} in the wind profile between about $z = 1.5$ and 2.5 km , which drives l_z^2 negative in the LLJ-composite profile (Fig. 5b).

In summary, the results from the simulations shown in Fig. 8 suggest that the presence of a low-level jet directed opposite to the motion of the bore is the key environmental factor supporting that bore. On the other hand, a surface-based layer of high static stability, topped by a deep layer of with lower N , as is often the case with nocturnal inversions, is not sufficient to support the bore.

5. The importance of finite amplitude

As discussed in section 3 the bore is not simply a linear trapped gravity wave. If these bores are supported by any type

of linear dynamics, it should be possible to isolate the bores and show that their structure and propagation are relatively insensitive to their amplitudes. For example, this was clearly evident in the case of the trapped waves shown in Fig. 4, where the structure and propagation of wave packets with maximum vertical velocities of 0.2 and 2 m s^{-1} were very similar. In the following, we test for amplitude independence in both the LLJ-composite-sounding and the ideal-jet, constant- N cases by isolating the bore from the triggering cold pool.

At the point in each dam-break simulation where the leading edge of the bore was at least 250 km in front of the cold pool, the velocity, pressure, and potential temperature perturbations about the undisturbed environment were extracted from a 200-km -wide region containing the bore. These perturbations were added back to the same undisturbed environmental base state and padded with buffers extending 100 km horizontally on each side of the bore throughout which the perturbations associated with the bore decayed exponentially to zero.

The bore isolated from the LLJ-composite-sounding dam-break simulation is shown at the initial time in Fig. 9a, and after 6 h of simulation in Fig. 9c, in which the windowed section of the full domain has been shifted to follow the leading edge of the bore. The amplitude and structure of the bore has remained almost steady during this period, as also evidenced by the trace of perturbation surface pressure plotted as a function of the horizontal coordinate at 0 and 6 h in Fig. 9, where once again the windowed region plotted at hour 6 has been shifted to follow the leading edge of the bore.

Results from an otherwise identical simulation in which the initial amplitudes are reduced by a factor of 10 are shown in Figs. 9b, 9d, and 9f, in which the vertical velocity field is contoured at $1/10$ the interval used in the full-amplitude simulation. At the initial time, the pattern in w is identical to that in the full-amplitude simulation, although the perturbations in w and the isentrope displacements are proportionately weaker. In contrast to the full-amplitude case, the bore in the $1/10$ -amplitude simulation dissipates rapidly. Just 1.5 h after it was initialized, there is essentially no trace of a horizontally propagating disturbance near the surface, and those perturbations that remain appear as vertically propagating gravity waves above the initial position of the bore (Fig. 9d). Similarly, the surface pressure perturbations lag far behind the 1.5-h position of the leading edge of the full-amplitude bore and consist of weak oscillations associated with the vertically propagating waves above the initial position of the disturbance (Fig. 9f).

A similar lack of amplitude independence occurs in the ideal-jet constant- N case. The perturbations in the w field, the isentropes and the surface pressure remain almost constant following the full-amplitude bore over a 6-h period, as shown in Figs. 10a, 10c, and 10e. When the perturbation amplitude is reduced by a factor of $1/10$, the disturbance essentially disappears over the first 1.5 h (Figs. 10b,d). Although a significant difference in the surface pressure across the windowed domain is still present at 1.5 h , the pressure gradient is much weaker and has remained at the initial position of the disturbance instead of propagating at the speed of the bore as in the full-amplitude simulation (Fig. 10f).

Given the strong amplitude dependence exhibited in these simulations of the isolated bore, and given that the environmental

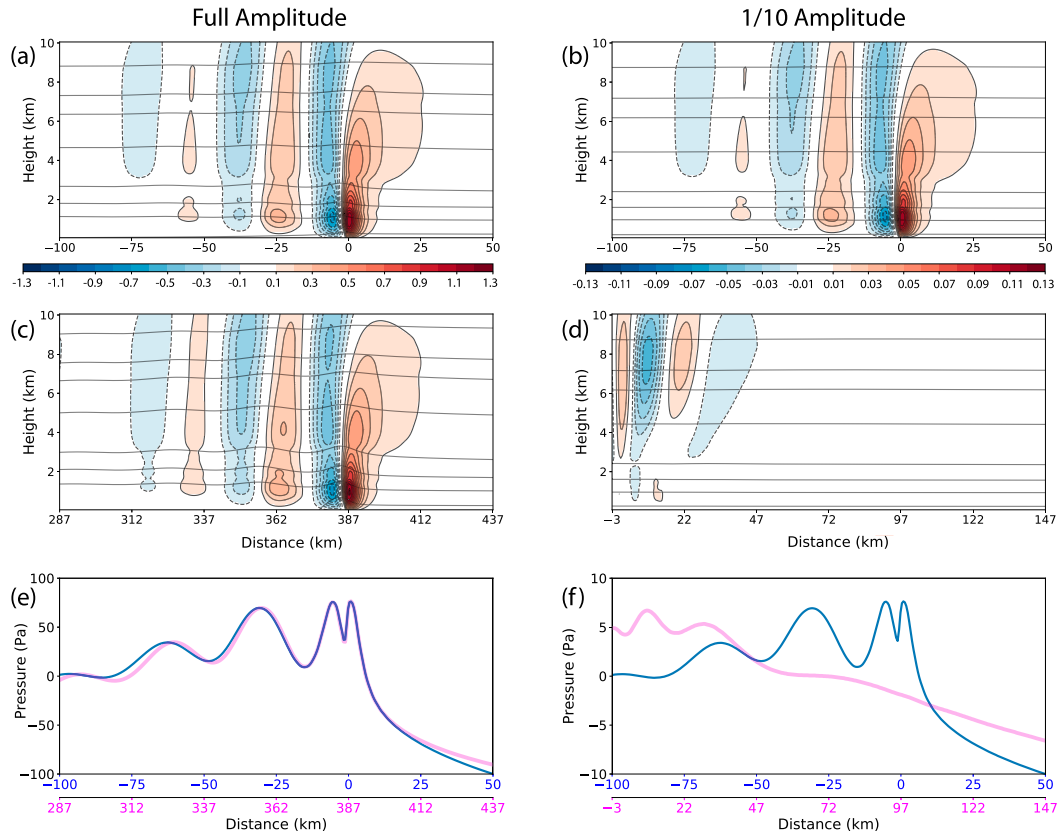


FIG. 9. Isolated-bore simulations for the LLJ-composite sounding. Vertical velocity (color contours; in m s^{-1}) and isentropes (contoured every 4 K beginning at 304 K). (left) Full amplitude, at hours (a) 0 and (c) 6. (e) Surface pressure perturbations for these times are superimposed, with the horizontal axis shifted to follow the bore. (right) Simulation with all perturbation amplitudes and the w -contour interval reduced a factor of 0.1 at hours (b) 0 and (d) 1.5. (f) Surface pressure perturbations for these times in the 0.1 case are superimposed.

sounding does not support trapped lee waves propagating in any direction close to that traveled by the actual bore, there is no obvious role for linear theory or Scorer-parameter analysis as a tool for understanding the long-time persistence of these bores.

6. The importance of dissipation

Turbulent mixing throughout the lowest 1 km at and behind the leading edge of the bore appears to play an important role in the bore dynamics. This mixing greatly reduces the low-level static stability behind the bore, as most easily seen by comparing the vertical spacing of the isentropes on each side of the bore in Fig. 9c, and noting that all air colder than 304 K has been warmed by vertical mixing after the passage of the bore. The relationship between the subgrid-scale TKE field and the surrounding bore structure is illustrated in Figs. 11a and 11c, which shows a windowed portion of the domain in the dam-break simulations for the LLJ-composite-sounding and constant- N ideal-jet cases at 3 h. In both cases there is a region immediately behind the leading edge of the bore in which the TKE exceeds $10 \text{ m}^2 \text{ s}^{-2}$, which is similar in magnitude and horizontal position to that obtained through numerical simulation in Koch et al. (2008a). The TKE decreases gradually

rearward of the leading edge, except near a local maximum underneath the first lee wave in the LLJ-composite-sounding case. In both cases, the top of the turbulent region is near the elevation of the wind-maximum in the low-level jet.

a. Sensitivity to grid resolution

Before further exploring the impact of subgrid-scale mixing in the dynamics of these bores, we checked the sensitivity of our results to grid resolution. To test this dependence we reduced the horizontal grid spacing from 500 to 200 m and repeated the dam-break simulations for the LLJ-composite-sounding and ideal-jet cases. The vertical grid spacing remained 100 m, implying that the finer-resolution grid is nearly isotropic. The results of these fine-mesh simulations are shown in Figs. 11b and 11d, which may be compared to the corresponding coarse-resolution results in Figs. 11a and 11c. The vertical velocities are just slightly stronger in the finer-resolution simulations, but overall, the w field and the isentropes are nearly identical, independent of resolution. There is some difference in the strength of the TKE at different resolutions, but this should be expected.

At finer resolution, the range of wavelengths that are subgrid scale is reduced, so the strength of the subgrid-scale TKE

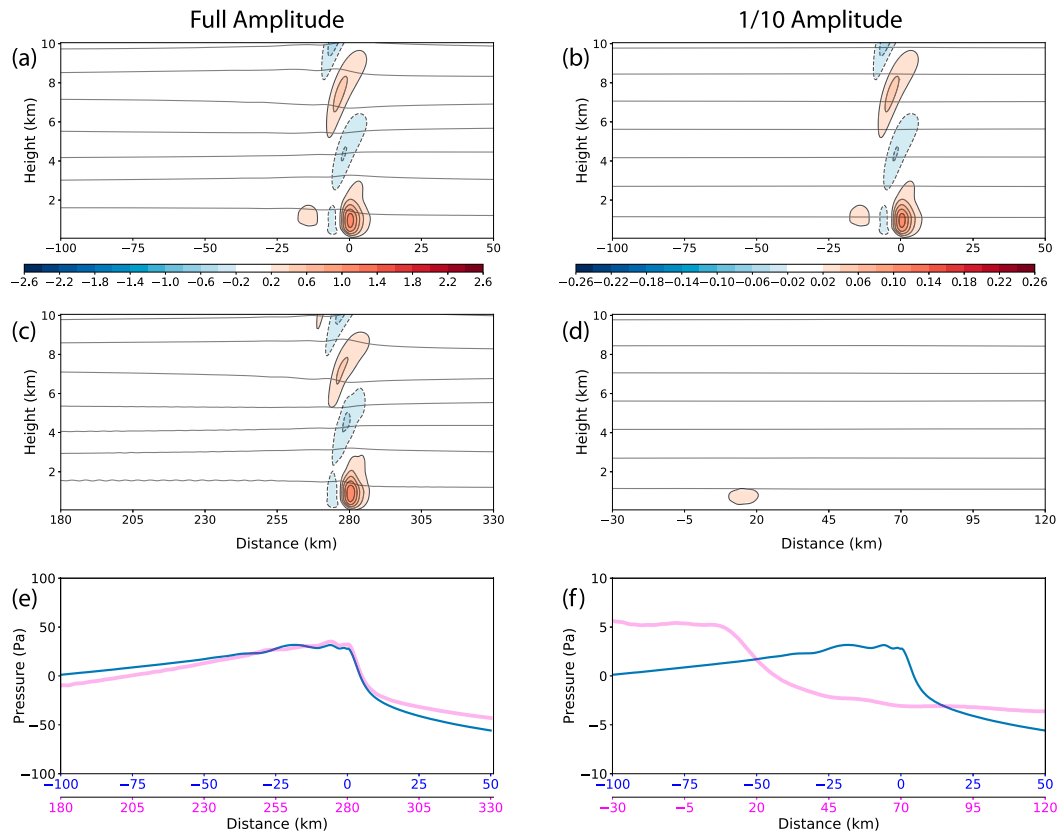


FIG. 10. As in Fig. 9, but for the ideal-jet, constant- N environment; isentropes are contoured at intervals of 10 K, starting at 310 K.

should decrease. In both cases, the ratio of the maximum TKE in the 500-m simulation to that in the 200-m simulation is approximately 1.25. As derived in Weyn and Durran (2017), if the kinetic energy spectrum of a disturbance is proportional to the wavenumber k to some power β , where $k = 2\pi/\lambda$ and λ is the wavelength, then the ratio of the kinetic energy in the range of wavelengths smaller than λ_1 to that in the range smaller than λ_2 is

$$R = \frac{\text{KE}(\lambda < \lambda_1)}{\text{KE}(\lambda < \lambda_2)} = \left(\frac{\lambda_1}{\lambda_2}\right)^{\beta-1}. \quad (5)$$

Choosing 5/2 as the ratio of wavelengths bounding the subgrid-scale cutoff in the coarse- and fine-mesh simulations, R will be approximately equal to the 1.25 result from the simulations when $\beta = 5/4$. Given the variations in the slope of the atmospheric kinetic energy spectrum about the average mesoscale value of $k^{-5/3}$ in both weather forecasts and idealized simulations (Selz et al. 2019; Menchaca and Durran 2019), the reduction in the strength of the maximum TKE in the fine-mesh simulations appears reasonable. Note that the region throughout which the TKE exceeds $2 \text{ m}^2 \text{ s}^{-2}$ is very similar at both resolutions.

b. Bernoulli function analysis

One helpful way to diagnose the influence of dissipation is by evaluating changes in the Bernoulli function along bore-relative trajectories through the disturbance. For disturbances that are

adiabatic, inviscid, and steady in a frame of reference moving with the bore at speed c , the compressible Bernoulli function,

$$B = c_p T + gz + \frac{1}{2}[(u - c)^2 + w^2], \quad (6)$$

is conserved following fluid parcels in the bore-relative reference frame (Gill 1982, p. 82), where c_p is the specific heat at constant pressure, T is temperature; their product $c_p T$ is the enthalpy, and gz is the geopotential.

Figure 12a shows isentropes, contours of subgrid TKE, and three bore-relative trajectories originating upstream at elevations of 100, 500, and 1500 m. The bore-relative velocity is from right to left, and the changes along each trajectory in B and its components are plotted in Figs. 12b–e, color-coded to match the colors of the trajectories in Fig. 12a. The Bernoulli function is almost perfectly conserved along the trajectory that originates at a height of 1.5 km (red dashed curves), although there are compensating changes in the potential energy and the enthalpy as air parcels rise and fall crossing the troughs and crests. This trajectory also closely parallels the isentropes.

In contrast, B starts to change along both of the lower trajectories as soon as they intersect the $2 \text{ m}^2 \text{ s}^{-2}$ contour for the subgrid TKE. The changes in ΔB along both of these trajectories are produced by small differences between two large terms: the potential energy and the enthalpy; the changes in kinetic energy are comparatively small. There is net ascent and

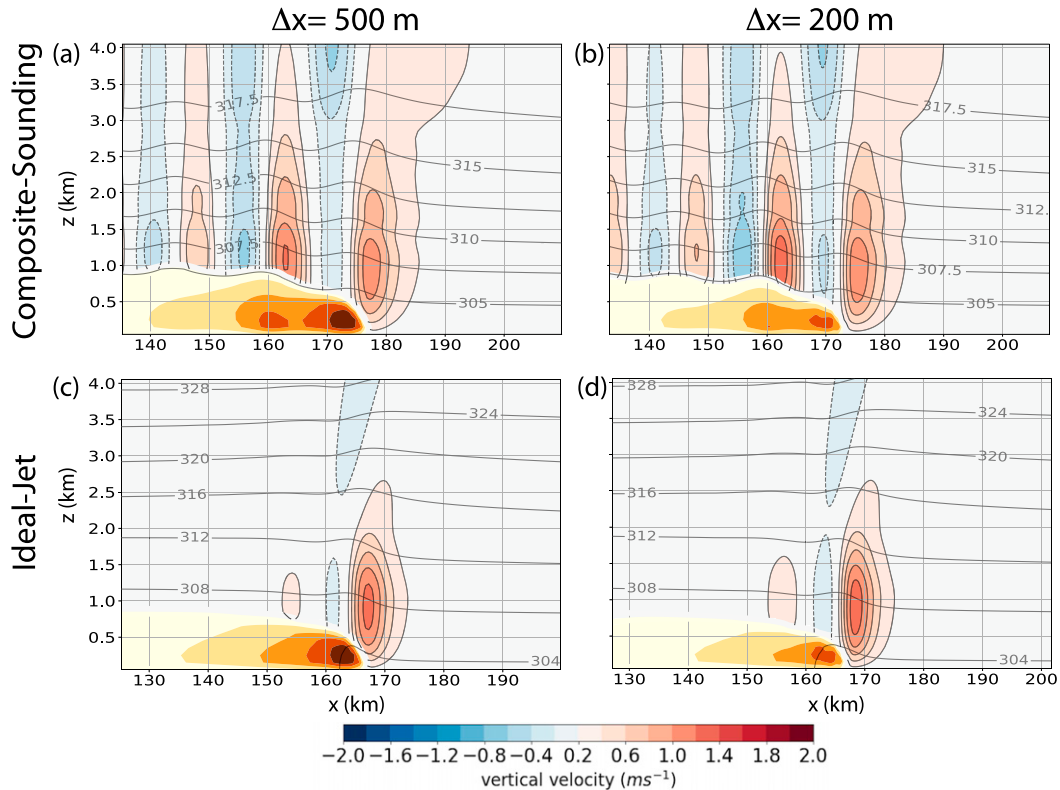


FIG. 11. Subgrid-scale TKE (yellow-to-brown color-filled contours starting at $2 \text{ m}^2 \text{ s}^{-2}$ and increasing at intervals of $2 \text{ m}^2 \text{ s}^{-2}$), vertical velocities (m s^{-1}), and isentropes of potential temperature at 3 h for the (a),(b) LLJ-composite-sounding and (c),(d) ideal-jet simulations. Horizontal resolution is (a),(c) 500 or (b),(d) 200 m.

also downstream warming along the lowest trajectory that produces a gradual increase in ΔB of about $1100 \text{ m}^2 \text{ s}^{-2}$. The trajectory that originates at $z = 500 \text{ m}$ experiences substantially more ascent than the lowest trajectory, but in the turbulent region this is more than offset by cooling which reduces the downstream value of ΔB to about $-600 \text{ m}^2 \text{ s}^{-2}$.

The same behavior is present in the Bernoulli function analysis for the ideal-jet constant- N case, plotted in Fig. 13. Air parcels originating from the 1.5-km level far upstream conserve their Bernoulli function while being lifted by the bore, as adiabatic cooling decreases the enthalpy to compensate for the increase in potential energy. Air parcels originating from lower levels undergo changes in Bernoulli function as soon as they encounter the region of subgrid-scale mixing. The value of B again increases along the lowest trajectory as the parcel ascends and experiences heating downstream of the leading edge of the bore, while B decreases along the trajectory originating at 500 m because diabatic cooling induces a decrease in enthalpy that exceeds the increase in geopotential.

c. Relation between turbulence intensity and updraft velocity

These changes in the low-level Bernoulli function illustrate the essential role of diabatic mixing in reducing the static stability behind the leading edge of the bore, thereby allowing a quasi-permanent upward displacement of the isentropes after its

passage. The strength of the mixing is strongly correlated with the strength of the vertical velocity maximum at the leading edge of the bore, as illustrated in Fig. 14, which shows the maximum vertical velocity (w_{max}) as a function of the maximum subgrid TKE (TKE_{max}) at successive times in the three dam-break simulations of propagating bores triggered by density currents. First consider the bore in the composite- θ ideal-jet case (Figs. 8i,j). This bore is nonsteady, amplifying slowly early in the simulation, rapidly around 5 h into the simulation and then more slowly again late in the simulation. Despite the irregularities in the rate at which the bore amplifies, a very strong linear correlation relation between w_{max} and TKE_{max} is apparent in Fig. 14, with $(w_{\text{max}}, \text{TKE}_{\text{max}})$ beginning near $(0.8 \text{ m s}^{-1}, 11 \text{ m}^2 \text{ s}^{-2})$ at 2 h in the simulation and increasing to $(2.7 \text{ m s}^{-1}, 21 \text{ m}^2 \text{ s}^{-2})$ at 8 h. The situation with the other two bores is different because they are quasi-steady, so the w - TKE_{max} curves are short and highly localized. Starting at 2 h, w_{max} in the ideal-jet constant- N simulation remains an almost constant 1.1 m s^{-1} while over the next 6 h TKE_{max} gradually increases by roughly 20% about an average value of $12 \text{ m}^2 \text{ s}^{-2}$. The LLJ-composite-sounding case generates very similar, if slightly less steady, values of w_{max} and TKE_{max} to those in the ideal-jet constant- N case.

d. Influence on the environmental profile

In all three simulations that supported a bore, the region of strong turbulence behind the bore is approximately the same as

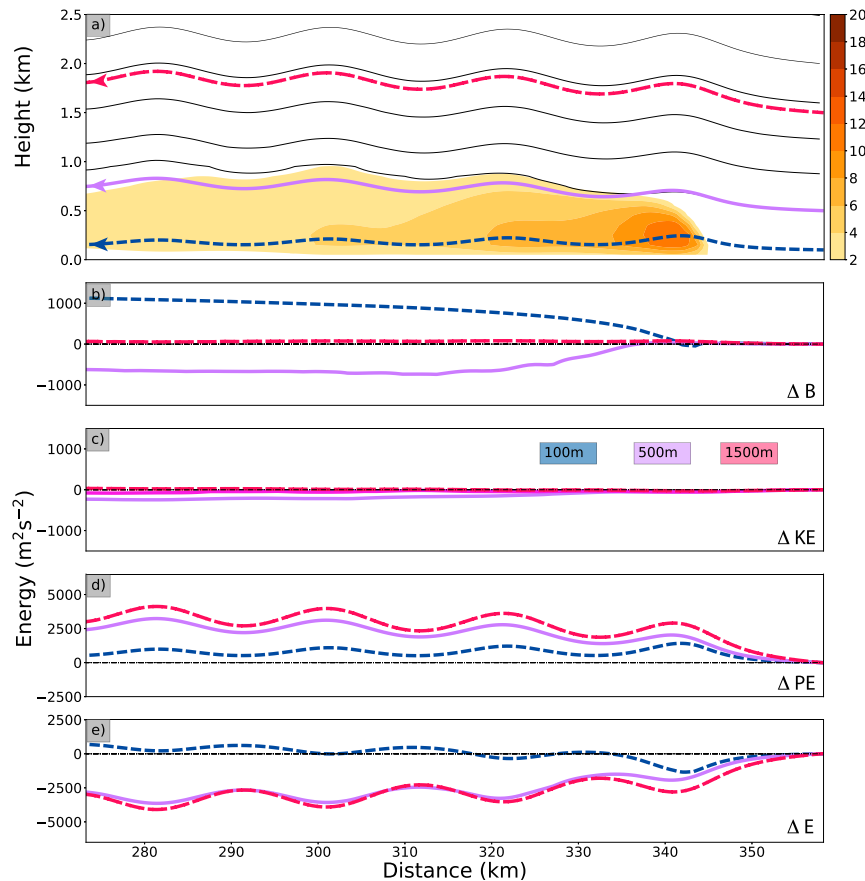


FIG. 12. Bernoulli analysis for the LLJ-composite-sounding simulation along trajectories starting ahead of the bore and traversing its circulation. (a) Trajectories (colored lines starting at elevations of 100, 500, and 1500 m), isentropes, and subgrid-scale TKE (color filled beginning at $2 \text{ m}^2 \text{ s}^{-2}$ and contoured every $2 \text{ m}^2 \text{ s}^{-2}$). Departures from the upstream values of the (b) Bernoulli function, (c) kinetic energy, (d) potential energy gz , and (e) enthalpy $E = c_p T$ along each color-coded trajectory.

that occupied by the trajectories of air parcels that were originally located below the jet maximum at $z = 600 \text{ m}$ in the upstream environment. This is apparent from the location of the purple trajectory, which originates at $z = 500 \text{ m}$, in Figs. 12a and 13a. The net influence of turbulent mixing on the vertical profiles of sensible temperature and horizontal wind speed is shown for the ideal-jet constant- N case in Fig. 15. These profiles are taken 100 km upstream and downstream from the leading edge of the bore. After the passage of the bore, surface temperatures warm and the lapse rate steepens, while the elevation of the jet maximum rises, roughly following an air-parcel trajectory. Note that long-wavelength internal-gravity wave perturbations extending 100 km upstream of the bore have induced minor modifications of the environmental winds.

7. Conclusions

We have investigated one prototypical nocturnal bore over the Great Plains through an extensive series of idealized numerical simulations. Our simulations are two-dimensional and

do not account for fully 3D bore structures propagating at some angle to the axis of the low-level jet. The restriction to two dimensions, our numerical resolution ($\Delta x = 200 \text{ m}$, $\Delta z = 100 \text{ m}$ at best), and the free-slip lower boundary condition limit our ability to faithfully simulate the turbulence accompanying the bore. Nevertheless, this two-dimensional framework is similar to that in most of the previous theoretical analyses of atmospheric bores, many of which are inviscid or include dissipation through a crude uniform eddy viscosity. Within these limitations in mind, our simulations suggest the following four conclusions.

Atmospheric-bore dynamics are inherently finite amplitude and have no obvious connection to linear internal wave dynamics. In particular, the bore is not propagating in an environmental wave duct formed by a decrease in the Scorer parameter with height. It should also be noted that wave packets propagate at their group velocity, not the phase speed of individual troughs and crests. Since the group velocity and phase speed are often quite different for short-wavelength trapped waves (7.5 and 15 m s^{-1} in the case shown in Fig. 4),

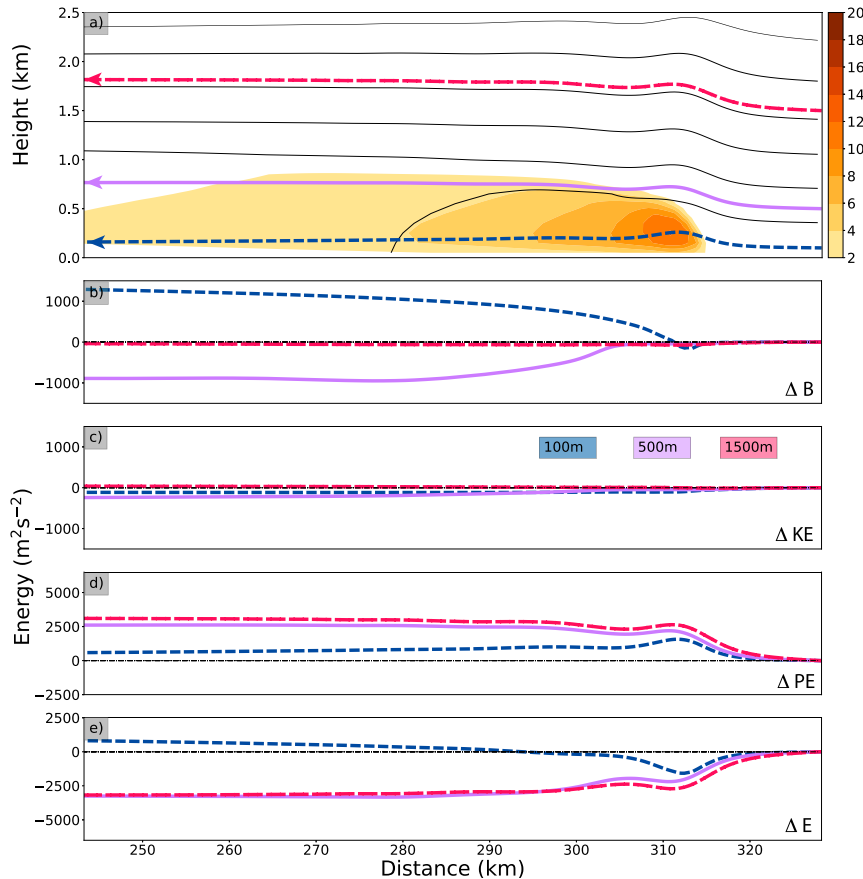


FIG. 13. As in Fig. 12, but for the ideal-jet environment.

if atmospheric bores were packets of trapped waves, the phase of the disturbance at the leading edge would be continually changing and observations might be expected to show as many bore passages marked by downdrafts as by updrafts.

A surface layer of high static stability, such as in a nocturnal inversion, topped by a deep layer of lower stability, is not essential for the development and propagation of the bore. A near-surface stable layer will nevertheless modify the bore structure and helps support the development of a series of waves downstream from the leading updraft. Near-surface stable layers may also be important in cases where only modest (5 m s^{-1}) low-level winds are opposing the bore (Clarke et al. 1981; Osborne and Lapworth 2017).

In these simulations, the key environmental condition necessary for the support of atmospheric bores is a strong low-level jet opposing the propagation of the bore. A bore with an updraft of 1 m s^{-1} is capable of quasi-steady propagation against this jet in an environment with constant stratification, leaving behind a shallow layer of reduced static stability. A weaker disturbance with the same spatial structure and 1/10 the amplitude does not propagate horizontally and dissipates completely within 1.5 h (Fig. 10).

Given the unimportance of low-level stability and the importance of a jet with strong vertical shear for supporting the bore, theoretical models for disturbances in shallow water appear to have little quantitative applicability to atmospheric

bores. Nevertheless, as in Durran (1986) the qualitative application of shallow-water theory may still be useful. In particular the bore represents a sudden transition to a deeper, more weakly stratified fluid state that is accompanied by

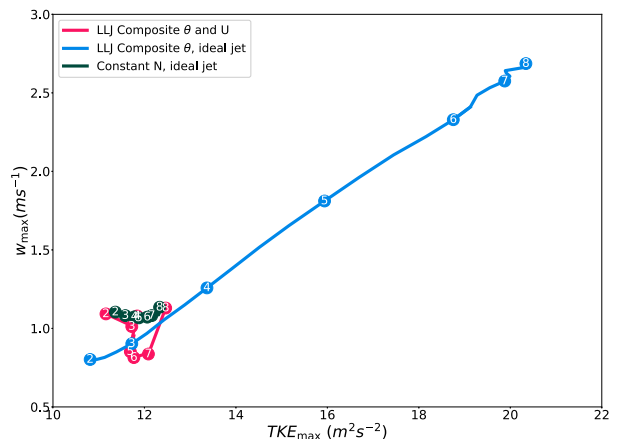


FIG. 14. Plot of maximum turbulent kinetic energy as a function of maximum vertical velocity for simulation environments consisting of the LLJ-composite sounding (red), the ideal-jet constant- N (green), and the LLJ-composite θ ideal-jet (blue) profiles. The numbers indicate time in hours elapsed in the simulation beginning at 2 h.

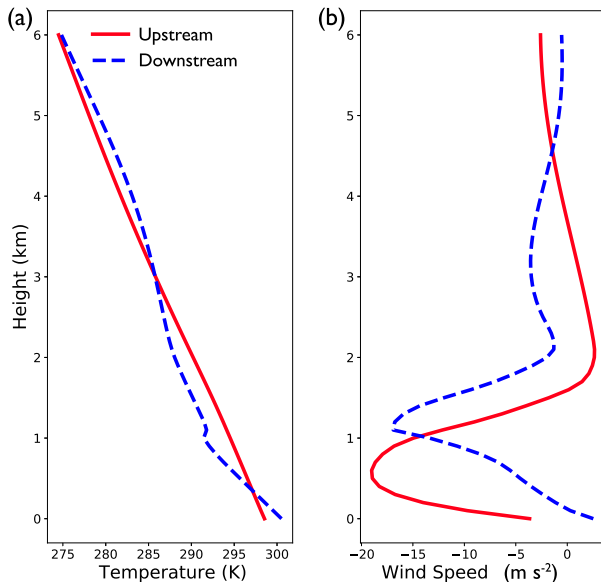


FIG. 15. Vertical profiles of (a) sensible temperature and (b) horizontal wind u in the ideal-jet constant- N simulation. Solid red (dashed blue) profiles are 100 km upstream (downstream) of the leading edge of the bore.

energy dissipation through turbulence and energy radiated away by waves, which in the atmospheric case, involves a combination of both vertically and horizontally propagating internal gravity waves.

As discussed in the introduction, it has long been recognized that atmosphere bores are accompanied by some turbulence. Clarke et al. (1981) may have provided the first observations of such turbulence, noting “there were some patches of light to moderate turbulence” as subjectively assessed during light-aircraft traverses of a morning glory on 29 September 1979. Nevertheless, except for very strong bores in which the layer depth jumps by a factor of 4 or more (Rottman and Simpson 1989), turbulence has not generally been seen as dominating the bore’s dynamics. The preceding simulations suggest a more central role for turbulence, in which the bore propagates into a low-level jet triggering shear instability in the layer between the jet maximum and the surface, creating a plume of turbulent fluid that continues to propagate away from the density current that initiated the bore.

In these simulations the jet Richardson number Ri_j (computed using the velocity change between the jet maximum and the surface to evaluate the shear) is 0.21 in the LLJ-composite sounding. Banta et al. (2003) found that such values of Ri_j beneath a strong nocturnal low-level jet in an undisturbed stable boundary layer were accompanied by shear-driven instabilities with TKE values between 0.2 and $0.6 \text{ m}^2 \text{ s}^{-2}$. In contrast to their undisturbed case, the strong updraft at the leading edge of the bores in these simulations appears capable of triggering much more intense shear-generated turbulence in the layer below the jet maximum. It would be interesting to repeat simulations such as that in Fig. 11d with a large-eddy simulation (LES) model to better analyze the dynamical

connections between the circulations at the leading edge of the bore and the intensity of the turbulence in its wake.

These simulations are based on a sounding representative of the conditions associated with one bore observed during the IHOP_2002 campaign, but at least one aspect of our results appears to generalize to a much larger set of 45 IHOP bore cases. Figure 11 of Haghi et al. (2017) shows that the wind shear vector that would be added to the low-level jet maximum to obtain the wind vector above the jet at either 1.5 or 2.5 km has a very strong tendency to be parallel to the propagation of the bore. The typical orientation of the wind shear vector in the layer above the jet core is therefore roughly similar to that in our 2D simulations as illustrated in Fig. 7b.

The shear vector between the surface and the jet maximum is not as frequently directly opposed to the bore in the IHOP data, but it still tends to have a component directed toward the bore. It may be all that is required between the surface and the level of the jet maximum is sufficient shear to support turbulence within that layer. We conducted additional 2D simulations (not shown) demonstrating that bores could be supported by weaker opposing jets than those in the preceding figures. In particular, a bore still occurs in the constant- N case with a weaker idealized jet profile in which all wind speeds are reduced by 50%. Further investigation is needed to better elucidate the relation between fully three-dimensional bores and the low-level winds.

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Data availability statement. All computations were performed using CM1. These two-dimensional simulations can be reproduced using minimal computer time, or alternatively the input files could be used as a departure point to vary the input parameters. Input data files for the model runs are archived with the University of Washington Libraries ResearchWorks repository: <http://hdl.handle.net/1773/45570>.

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