GFD II: Balance Dynamics
ATM S 542

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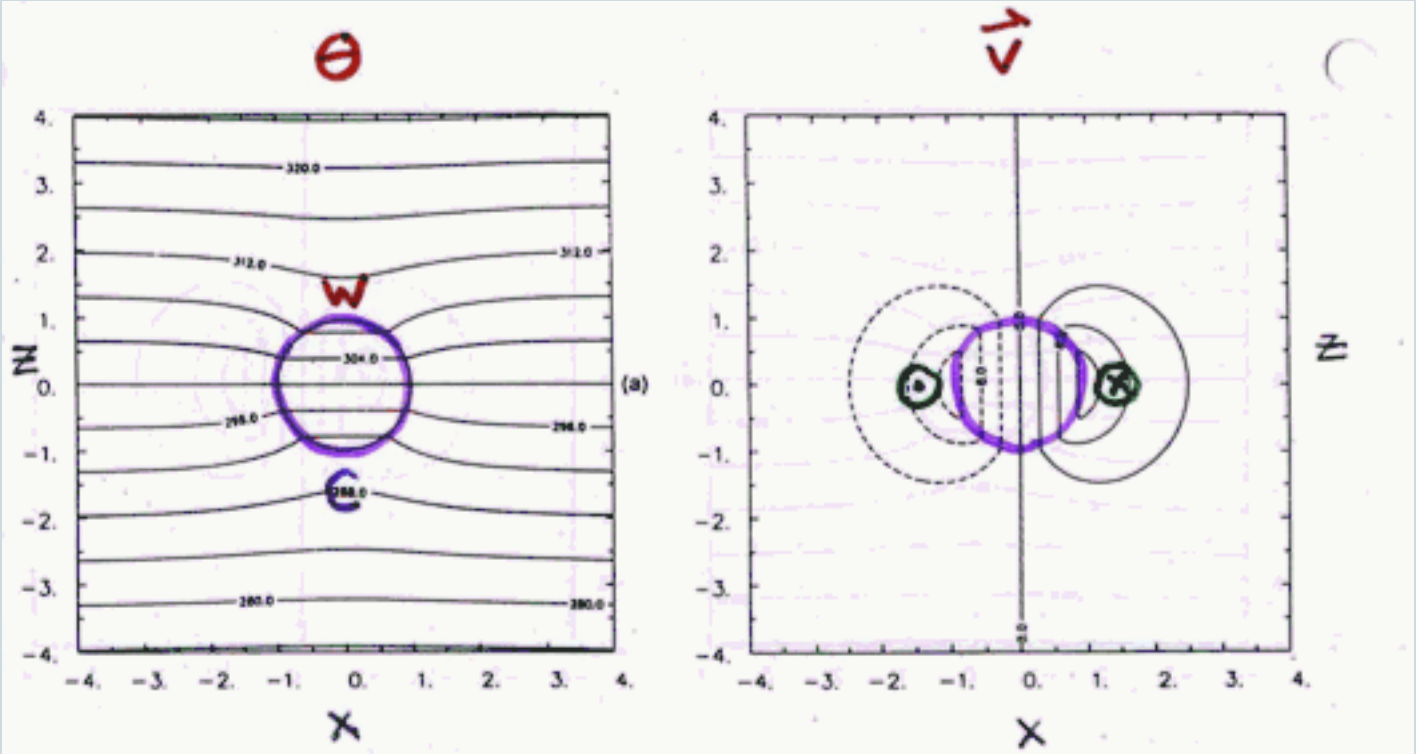
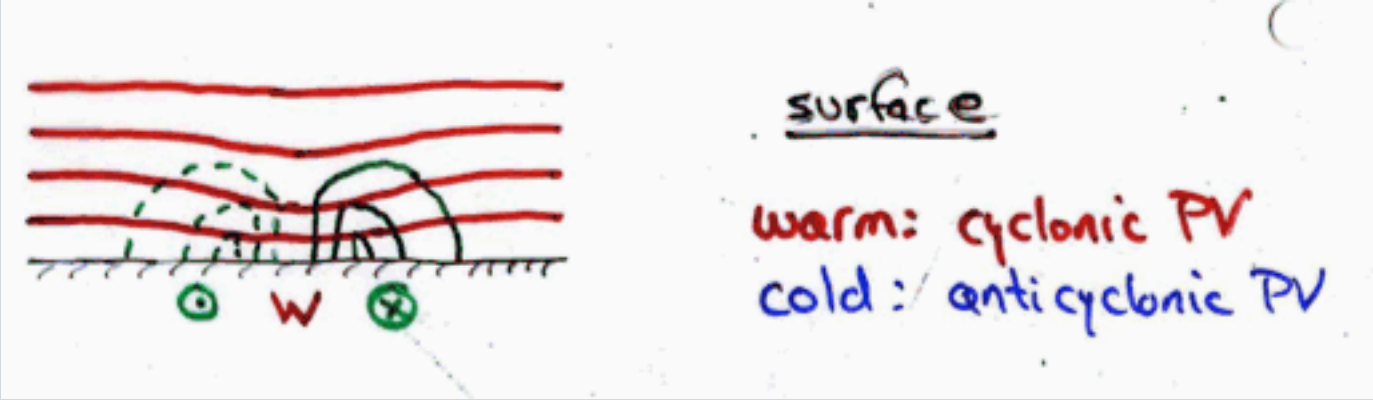
QG Equations

- The QG system
  - Started with 5 equations for 5 unknowns
  - Reduced to 1 equation for 1 unknown (streamfunction)
  - It's nonlinear though (so has very rich behavior)
  - Balanced in that there are no gravity waves

Response to a ball of PV



Response to a surface temperature anomaly



Just like top half of the previous picture!

PV Thinking

- Boussinesq, f-plane, adiabatic, frictionless QG PV equation:

$$\frac{D_g q}{Dt} = 0$$

  - Advection by geostrophic velocities
- Inversion:

$$q = \nabla^2 \psi + \frac{f_0^2}{N^2} \frac{\partial^2 \psi}{\partial z^2}$$

  - Can invert PV to get all the geostrophic quantities:

$$\psi \text{ (and } u_g, v_g, \zeta_g, b)$$

Ageostrophic velocities

- Any change in the individual quantities happens due to ageostrophic motions though:

$$\frac{Du_g}{Dt} = f_0 v_a \qquad \frac{Dv_g}{Dt} = -f_0 u_a$$

$$\frac{D\zeta_g}{Dt} = f_0 \frac{\partial w}{\partial z} \qquad \frac{Db}{Dt} = -N^2 w$$

Omega Equation

- Can reconstruct w from geostrophic quantities with a diagnostic relation: the omega equation
  - Derive this by eliminating partial derivatives with respect to time in the buoyancy and vorticity equation

W-thinking

- Thinking about factors that change the components of **thermal wind balance** leads to a completely different way of interpreting the ageostrophic velocities
- Geostrophic velocities are constantly trying to break apart thermal wind balance
  - Changing each the buoyancy gradient and shear in equal magnitudes, in the wrong direction to cancel

W-thinking

- Ageostrophic velocities are quietly working in the background to keep the flow in thermal wind balance
- This derivation also suggests a better version of the "omega equation"
  - "Q-vectors": best way to depict vertical motion

$$\mathbf{Q} = -\left(\frac{\partial \mathbf{u_g}}{\partial x} \cdot \nabla b, \frac{\partial \mathbf{u_g}}{\partial y} \cdot \nabla b \right)$$

and $$\nabla \cdot \mathbf{Q} = \frac{N^2}{2} \left(\nabla^2 w + \frac{f_0^2}{N^2} \frac{\partial^2 w}{\partial z^2} \right)$$

Q-vectors point towards ascent