GFD II: Balance Dynamics
ATM S 542

DARGAN M. W. FRIERSON
UNIVERSITY OF WASHINGTON, DEPARTMENT OF ATMOSPHERIC SCIENCES

4/5/11

Kelvin-Helmholtz Instability

- Courtesy of Greg Hakim

Barotropic Instability

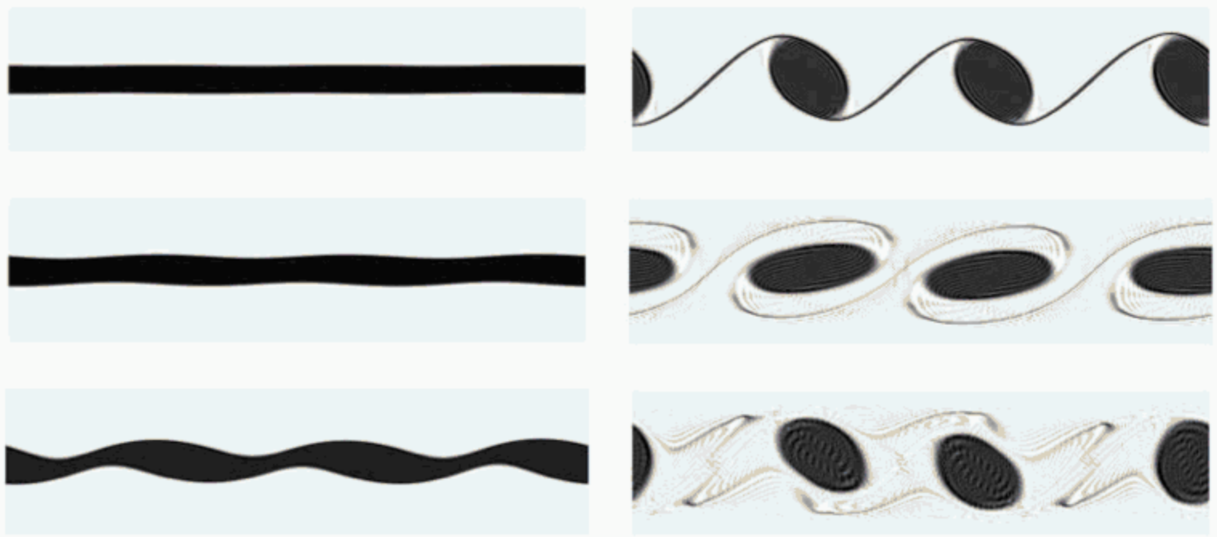
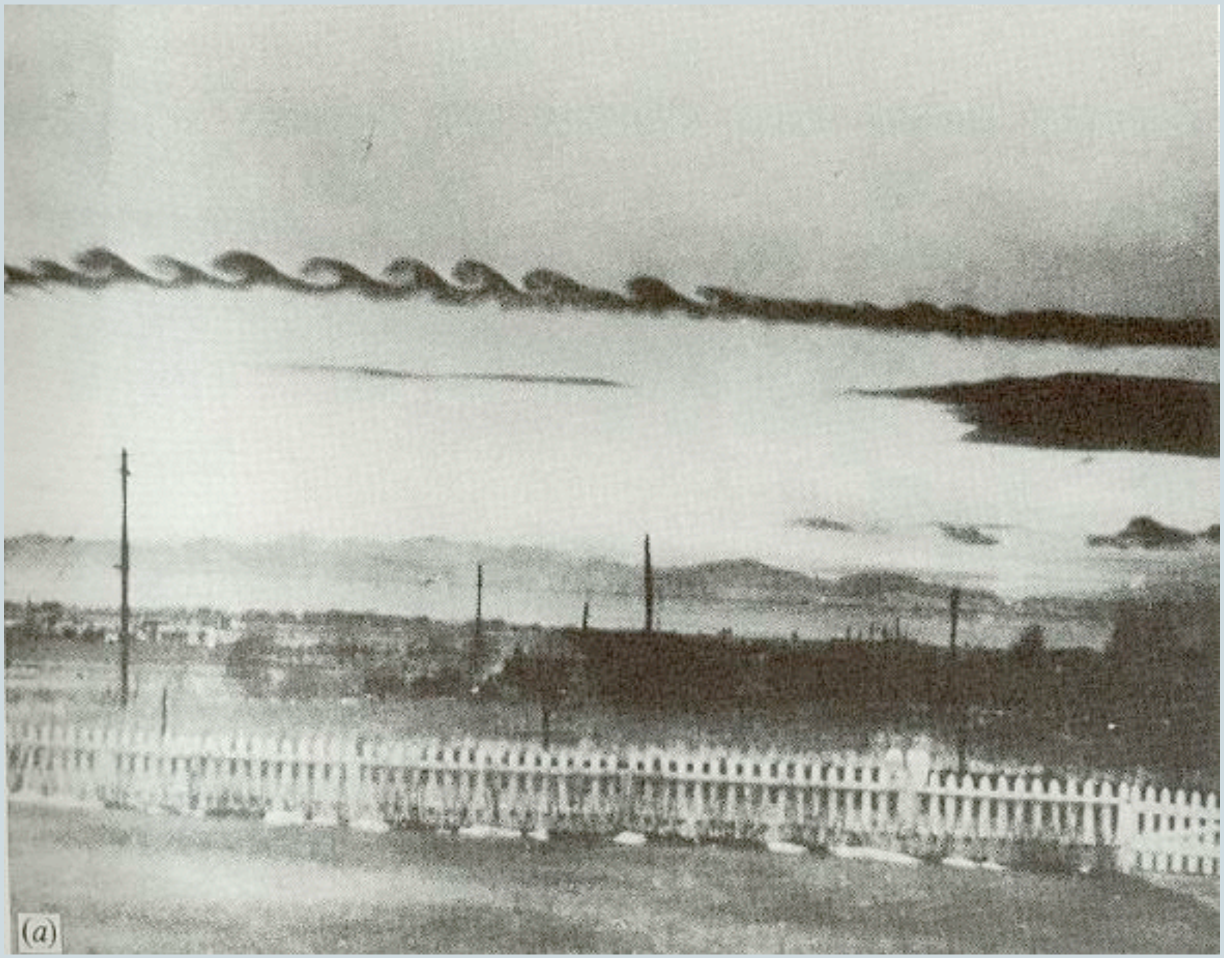


Fig. 6.6 A sequence of plots of the vorticity, at equal time intervals, from a numerical solution of the nonlinear vorticity equation (6.12), with initial conditions as in Fig. 6.4 with $a = 0.1$, plus a very small random perturbation. Time increases first down the left column and then down the right column. The solution is obtained in a rectangular (4×1) domain, with periodic conditions in the x-direction and slippery walls at $y = (0, 1)$. The maximum linear instability occurs for a wavelength of 1.57, which for a domain of length 4 corresponds to a wavenumber of 2.55. Since the periodic domain quantizes the allowable wavenumbers, the maximum instability is at wavenumber 3, and this is what emerges. Only in the first two or three frames is the linear approximation valid.

Kelvin-Helmholtz Instability

Kelvin-Helmholtz Instability



Kelvin-Helmholtz Instability

Barotropic Instability

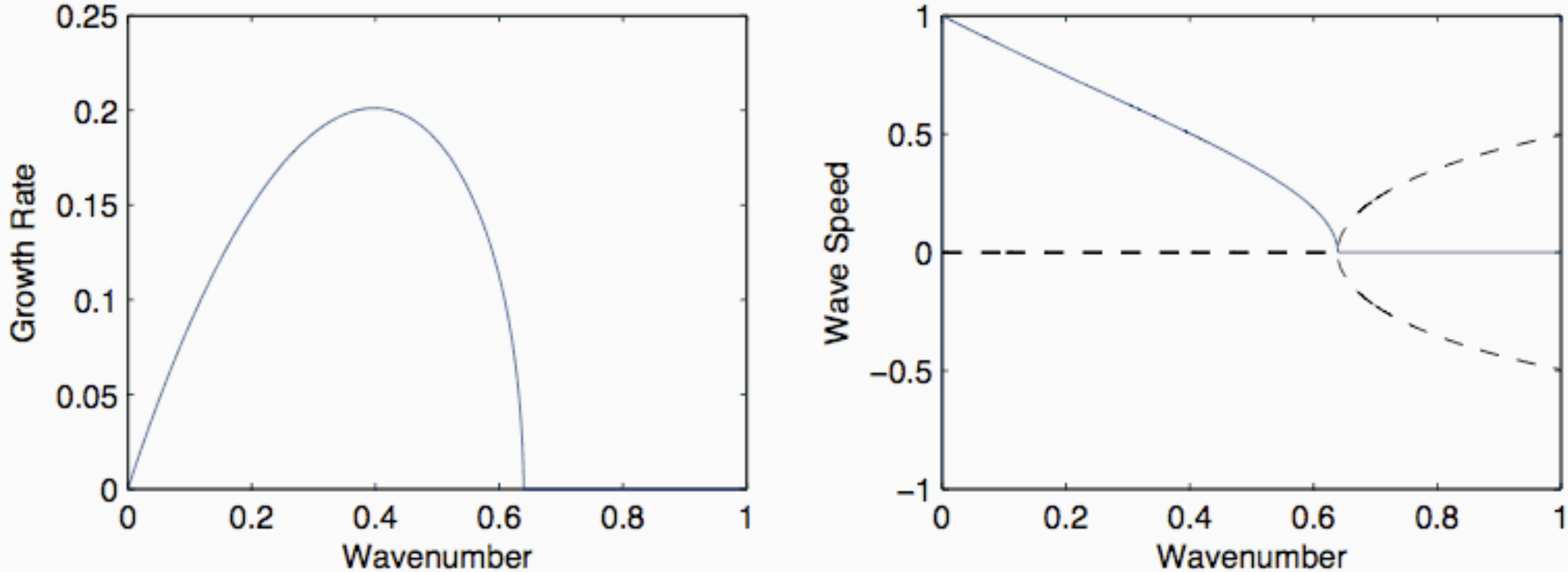


Fig. 6.5 Left: Growth rate ($\sigma = kc_i$) calculated from (6.42) with c non-dimensionalized by U_0 and k non-dimensionalized by $1/a$ (equivalent to setting $a = U_0 = 1$). Right: Real (c_r, dashed) and imaginary (c_i, solid) wave speeds. The flow is unstable for $k < 0.63$, with the maximum instability occurring at $k = 0.39$.

Barotropic Instability

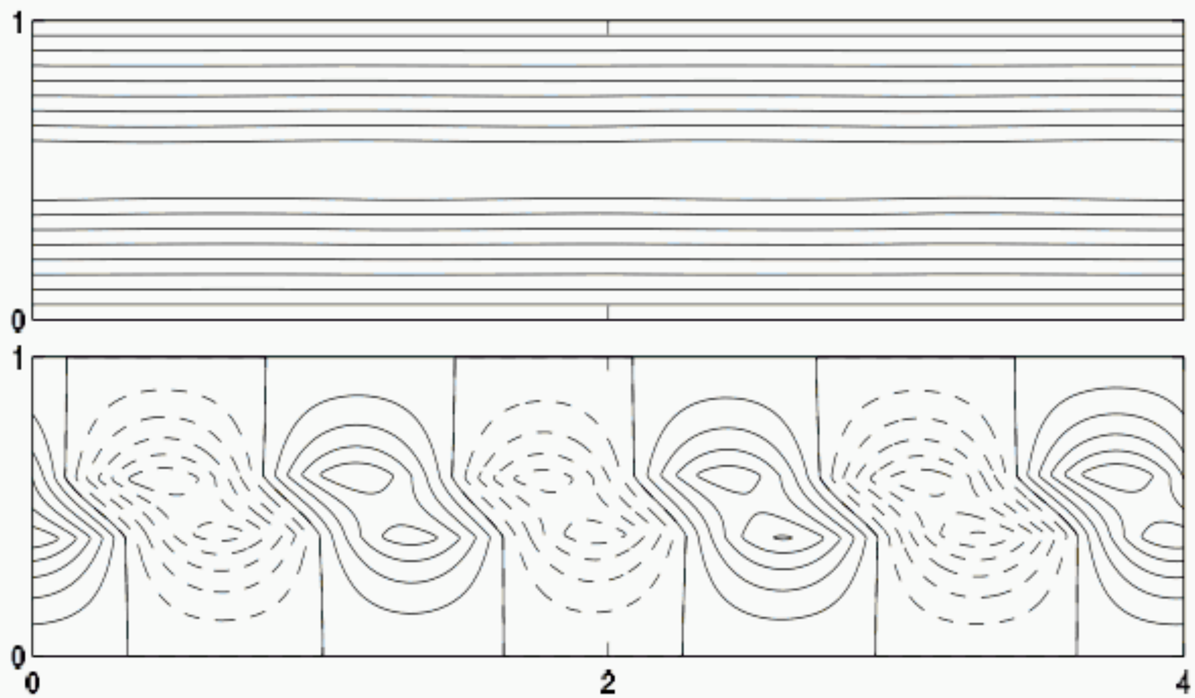
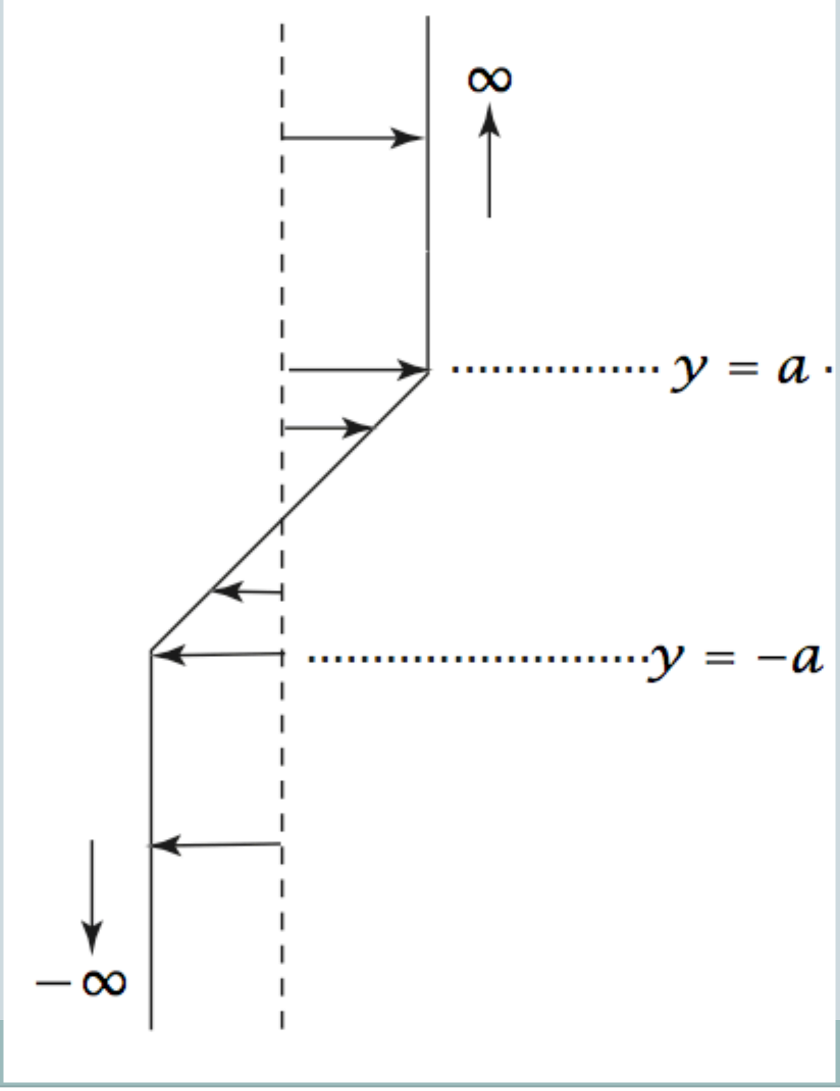


Fig. 6.7 The total streamfunction (top panel) and the perturbation streamfunction from the same numerical calculation as in Fig. 6.6, at a time corresponding to the second frame. Positive values are solid lines, and negative values are dashed. The perturbation pattern leans into the shear, and grows exponentially in place.

Barotropic Instability

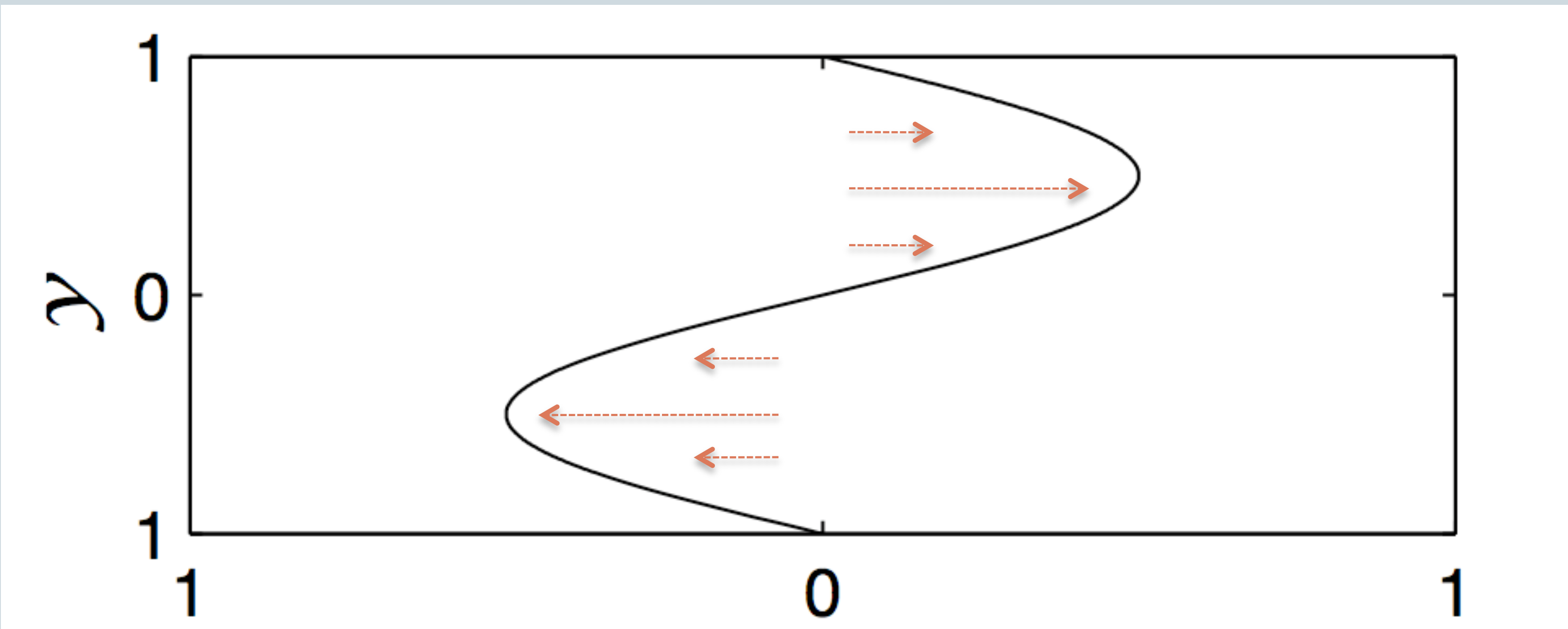
- We studied this:



Probably reasonable to assume that smoothed versions of this profile would also be unstable…

Barotropic Instability

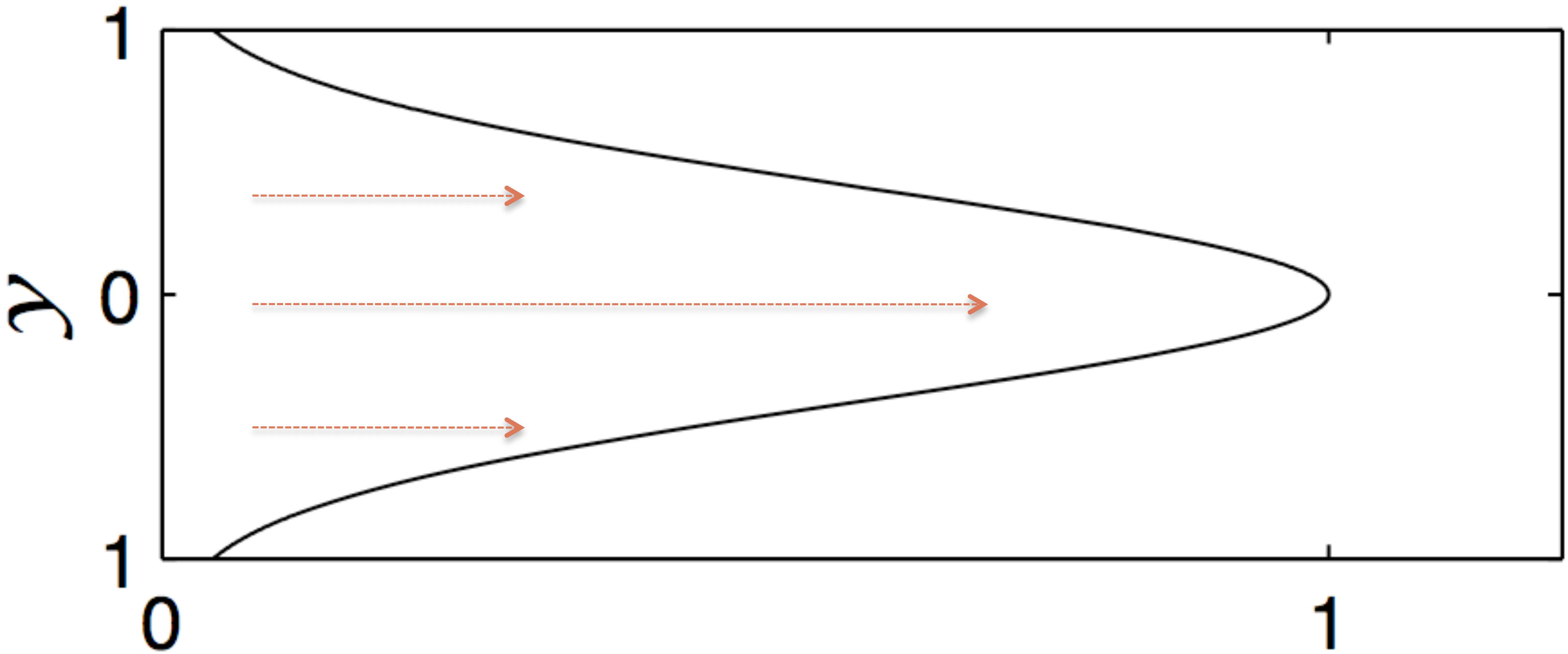
- But how about this profile?



Velocity doesn't stay constant at peaks…

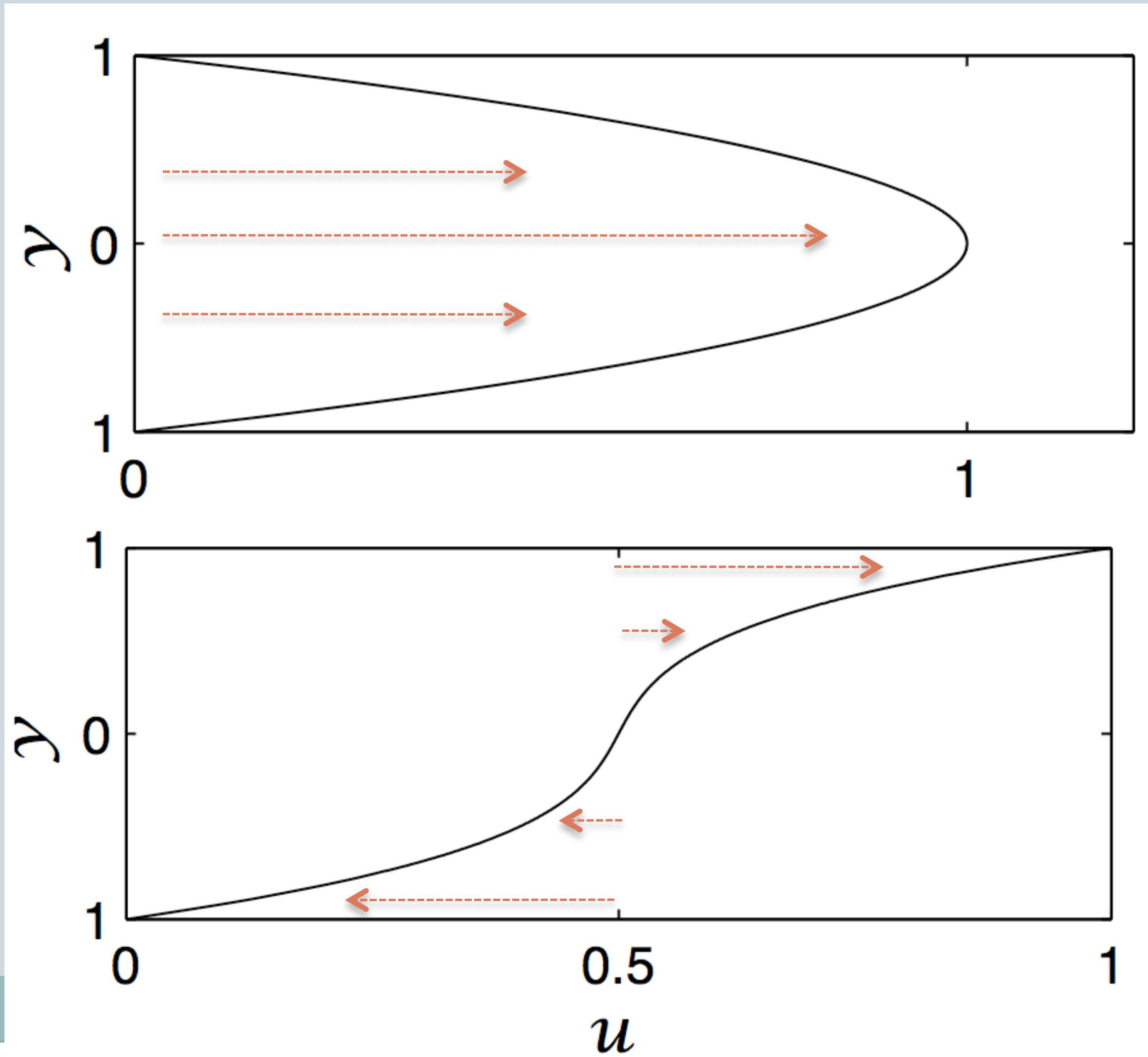
Barotropic Instability

- Or this profile?



Kind of looks like our example in the bottom half…

Barotropic Instability



Or these?

Both have shear, when can that be tapped by eddies?

Barotropic Instability

- We'll derive two criteria for barotropic instability:
  - Rayleigh's criterion
  - Fjortoff's criterion
  - Former has to do with counter-propagating Rossby waves, latter has to do with zonal wind speeds at location of different waves
- These are *necessary* conditions but not *sufficient* conditions
  - There is no way to say there will *definitely* be barotropic instability in any given situation without solving the full problem
  - Criteria are extremely useful in practice though

Back to the Profiles from Before:

Zonal winds

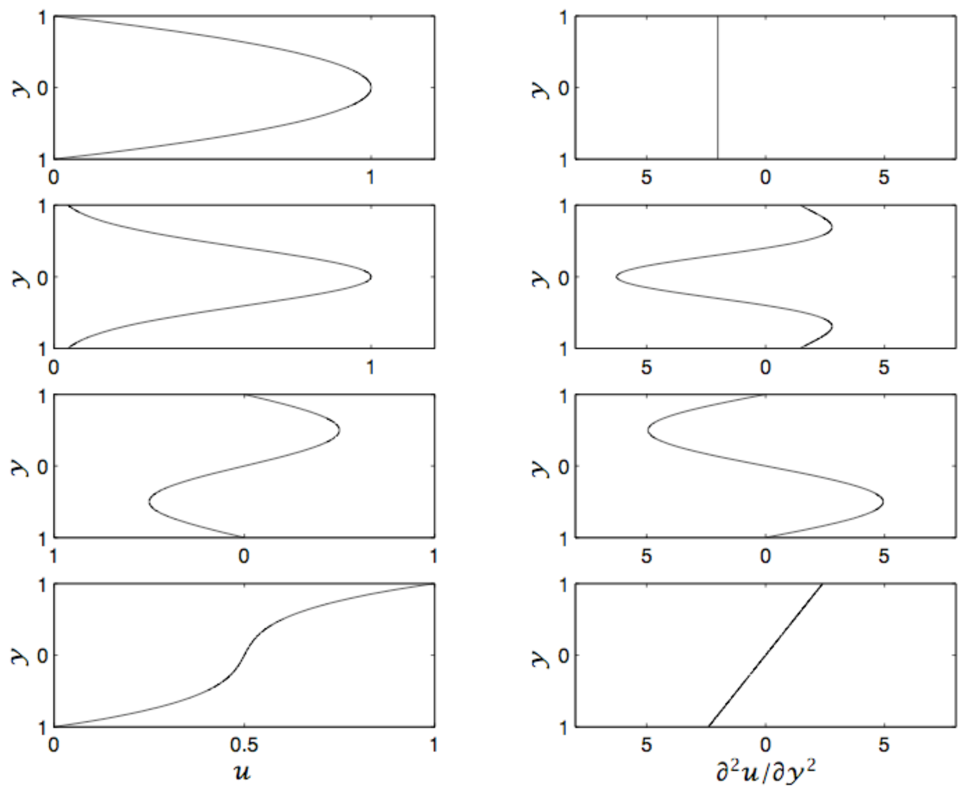


Fig. 6.8 Example parallel velocity profiles (left column) and their second derivatives (right column). From the top: Poiseuille flow ($u = 1 - y^2$); a Gaussian jet; a sinusoidal profile; a polynomial profile. By Rayleigh's criterion, the top profile is stable, whereas the lower three are potentially unstable. However, the bottom profile is stable by Fjørtoft's criterion (note that the vorticity maxima are at the boundaries). If the β-effect were present and large enough it would stabilize the middle two profiles.

d^2 u
-----
dy^2