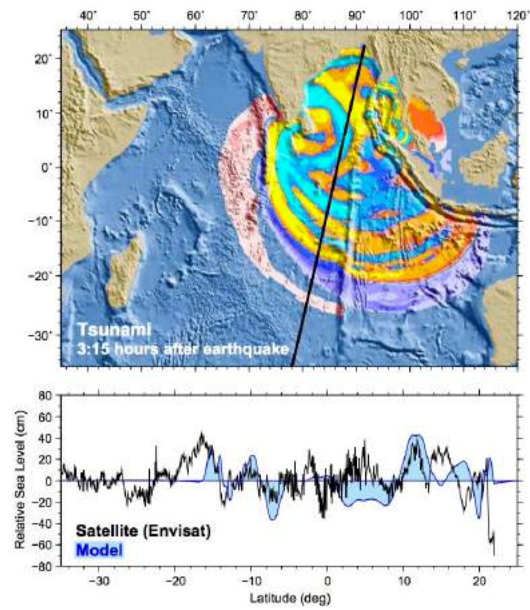
GFD1 AS509/OC512 Winter Q. 2017 Dargan Frierson
Lab 6 P.B.Rhines Kelvin Waves and other inertial/gravity waves Tues 7 Feb 12.30

With this lab we return to wave propagation; succeeding labs will look at internal gravity waves, lee waves, Rossby waves and their relationships with general circulation. The sequence of ideas begins with classic gravity waves in a uniform-density (unstratified) fluid. There is a sharp division of propagation between dispersive (short-wavelength, deep water, non-hydrostatic pressure field) and non-dispersive (long-wavelength, possibly shallow water, hydrostatic).

Consider the hydrostatic, long-wave case. These waves are visible as a tsunami response to earthquakes, propagating at its $\sqrt{gh}$ speed, ~ 300 m sec^{-1}, across the world. Aside from slowing and refracting above changes in ocean depth h, it is essentially non-dispersive. This keeps its energy from spreading out, leading to the devastating encounters when it comes into shore, where its energy is squeezed into a smaller depth and smaller horizontal distance as the wavelength decreases … raising its amplitude hugely.



The 2004 Sumatra tsunami hitting the coast of Thailand. Probably the first crest to arrive although this is not certain. Note how the seawater has drained out of the bay in advance of the major waves. Some wave dispersion developed as it came into shore (multiple wave crests). An altimetry satellite ERS1 flew over (along the black track above) at 3:14 hours after the earthquake, showing mid-ocean surface height range ~ 40 cm, in fair agreement with the numerical simulation shown above, right. While this seem innocently small, the 300 m sec^{-1} propagation speed means that the wave energy transport (energy density x group velocity) is large.

the dispersion relation, and also a new term expressing the coupling between the wave equation and the potential vorticity field. The longest wavelengths have a frequency σ just above f, and no free waves are possible below f: a wave
$\eta = A \sin(kx + ly - \sigma t)$ has a dispersion relation

$$\sigma^2 = f^2 + gh(k^2 + l^2).$$

Inertial oscillations of a particle on a rotating planet (or a fluid ideally moving like a rigid horizontal sheet) are essentially this long-wave limit. These are now dispersive waves, with group velocity c_g less than the phase speed : $c_g \equiv (\partial\sigma/\partial k, \partial\sigma/\partial l)$, the slope of the dispersion curve $\sigma(k, l)$. Thus the longest waves have near zero group velocity, again suggesting why we see near-inertial oscillations hanging around as geostrophic adjustment takes place.

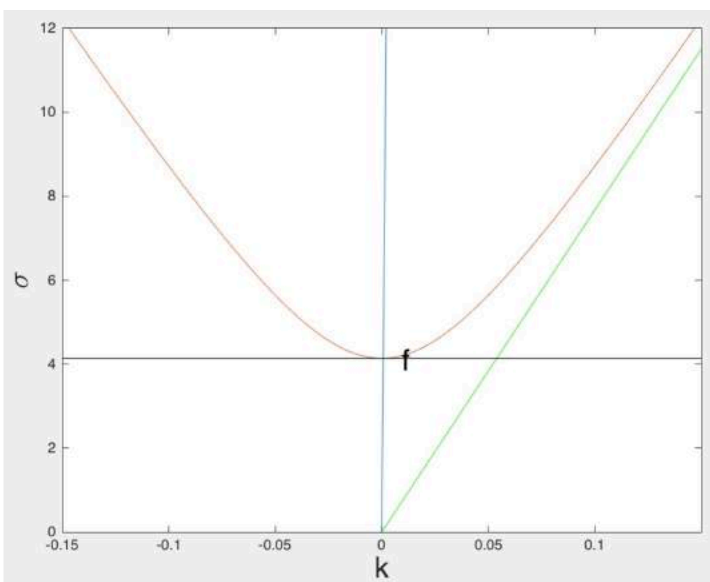
With no free long waves at frequencies below f, we wonder how ocean tides behave (for those are long waves, with low frequency components like the 24 hr solar diurnal tide, with frequency = Ω). To understand the Kelvin wave, imagine a narrow water channel (like the one in the GFD lab) on a rotating Earth. Ordinary long gravity waves can propagate along this channel, despite the Coriolis force acting on the along-channel orbital velocity of fluid parcels. The waves simply 'lean on' the two vertical boundaries. The Coriolis force is transmitted to the boundaries by a slight tilt (across-channel) in the free surface, supplying a pressure gradient. The dispersion relation does not depend on f:

$$\sigma^2 = gh\, k^2 \,.$$

where k is the wavenumber along the channel. The remarkable thing is that you can make the channel infinitely wide and still find this wave leaning on a single boundary.

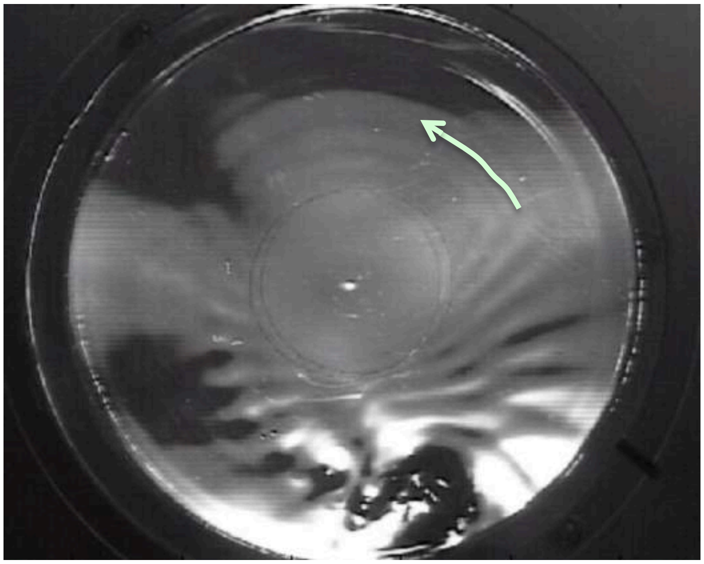
Lateral boundaries introduce a new family of long waves at all frequencies, an 'edge wave' or 'coastally trapped wave' mode exists, in which η varies like a sine wave along the coast (let that be the x-direction), yet is trapped near the coast with exponentially decreasing amplitude normal to the coast (in the y-direction). The basic theory assumes a straight coastal boundary, a vertical wall.

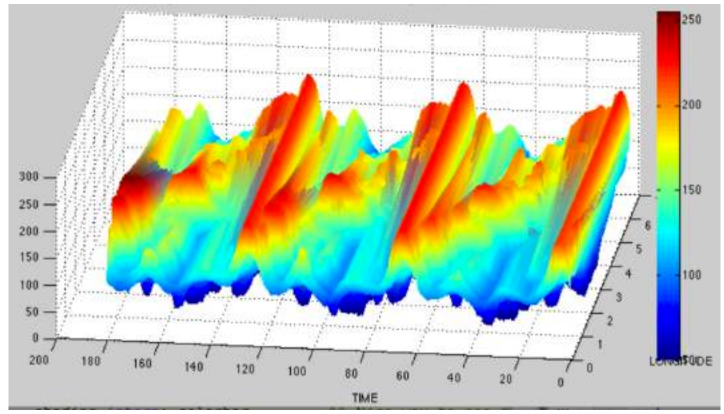
This 'Kelvin wave' is organized with vanishing v-velocity normal to the coast, and u-velocity (along the coast, in the x-direction) in geostrophic balance with the offshore pressure gradient, $\rho g\, \partial\eta/\partial y$. The along-shore momentum equation is simply that of a non-rotating fluid since the x-Coriolis force, fv, vanishes everywhere and thus the dispersion relation of the Kelvin wave is identical to that of the long, non-dispersive, zero-f, gravity wave. This balance works only for propagation in one direction: think of it as the cyclonic direction around an ocean basin, counter-clockwise in the Northern Hemisphere. That is, propagation to the right, looking offshore in the northern hemisphere.

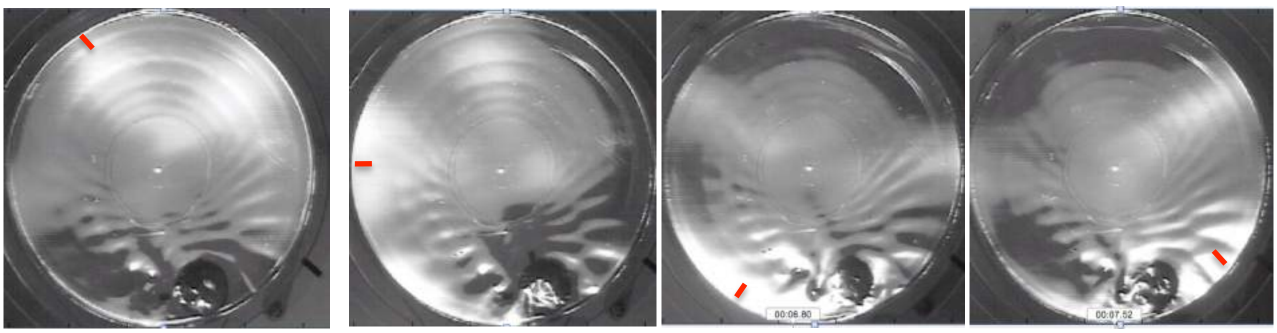


Dispersion relation for hydrostatic, long inertial-gravity waves (red, with y-wavenumber l = 0) and the Kelvin wave (green). k is the wavenumber along a coast (positive k is propagation with the coast lying to the right in northern hemisphere).

The experiment in the figures below show the Kelvin wave propagating very rapidly round the cylinder, with amplitude that is greatest near the outer wall.





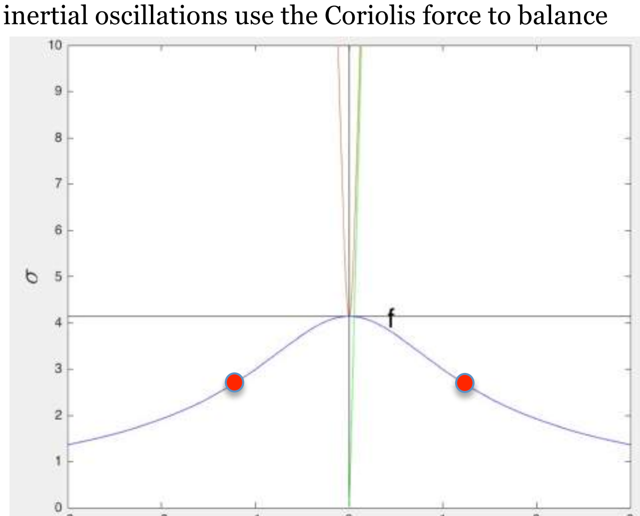


Upper Left: Kelvin wave in the 1.3m diameter rotating cylinder, seen from above. Mean water depth 6 cm. Ω = 2.1 hence f = 2.2 sec^{-1}. Wave phase speed √(gh) = 75 cm sec^{-1}. The wave is excited by oscillating the rotation rate sinusoidally at frequency σ = 1.25 sec^{-1} which is roughly ½ f (under computer control); this is in the presence of a small mountain at 6 o'clock. The arrow shows the phase propagation. The short waves are internal inertial waves also generated at frequencies below f. A third response to sloshing fluid over a mountain is cyclonic eddies and these are sort of visible in the chaos near the mountain. **Right**: An x-t diagram (Hovmoeller plot) of the wave amplitude vs. longitude (y axis) and time (x axis). The dominant sloping ridge propagates while passing through the forcing region which has a stationary eddy signal, and short inertial waves propagating much more slowly. **Lower:** time sequence showing cyclonic propagation. Red ticks mark a Kelvin-wave crest. See the movie at http://www.ocean.washington.edu/research/gfd/oc512-gfd1-2017/

The short wave crests in the images are internal waves, which like inertial oscillations use the Coriolis force to balance acceleration. They live dominantly at much shorter horizontal wavelength than hydrostatic Kelvin waves and long inertial gravity waves. Their dispersion relation is

$$\sigma^2 = \frac{f^2 m^2}{k^2 + m^2} = f^2 \sin^2\theta$$

for a wave with pressure varying like p = A sin (kx + mz – σt) where θ is the angel between the wave vector (k,m) and the horizontal. Units of k and m are cm^{-1}. This dispersion diagram shows how great is the length-scale separation between Kelvin waves, hydrostatic inertial-gravity waves and inertial internal waves. The red dots show the frequency and wavenumber of the internal waves excited in the GFD lab experiment (for the 'longest'

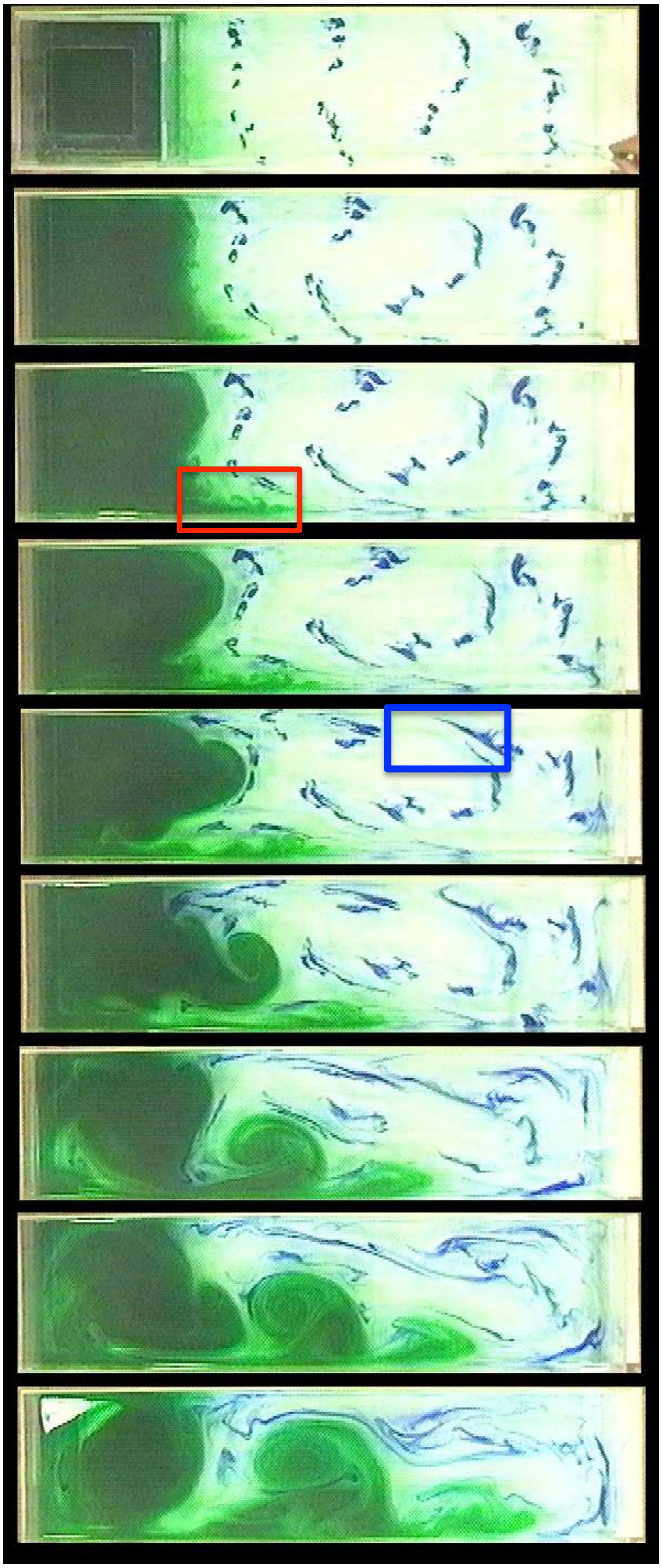


vertical wavelength, $m = 2\pi/6\ cm^{-1}$ and $k = 1.25\ cm^{-1}$). The Quicktime video of these waves can be found at http://www.ocean.washington.edu/research/gfd/oc512-gfd1-2017/ .

Density stratification. We introduce these non-hydrostatic internal waves here because of their great similarity with internal *gravity* waves in a stratified fluid. In addition, Kelvin waves exist in a density-stratified fluid. In fact much of what you learn from the 1-layer wave equation generalizes to a density stratified atmosphere or ocean rather simply, so long as the mean depth of the fluid is uniform; a sloping bottom introduces new wave modes that resemble Rossby waves.

In the lab we made a 1 ½ layer stratification (a thin layer of fresh water floating on top of a thick layer of salty water). As with non-rotating long gravity waves, Kelvin waves exist in this fluid, with g simply replaced $g' \equiv g\Delta\rho/\rho$. While the demonstration did not clearly show linear Kelvin waves, a 'blob' of fresh water injected at the top propagated as a sort of boundary jet, cyclonically around the basin. This introduces the idea that Kelvin waves can actively express the development of fluid circulation around the boundary.

The figure sequence below shows a more careful experiment with a rectangular channel on the rotating table, and a 1 ½ layer stratification. The green dyed fluid at the left is in the upper layer, and is held back by a vertical barrier. The upper layer is thicker where the green dye lies. When the vertical barrier is suddenly removed the extra fluid in the upper layer surges along the boundary, following the Kelvin wave pathway. In addition a fast linear Kelvin wave has propagated ahead of the dyed water mass, as seen by the movement of the blue lines of dye. This is a subtle situation in which waves are 'forerunners' of a general circulation of the fluid. Along the West Coast of the US atmospheric Kelvin waves are visible in a low-level maritime layer of air which surges northward along the mountainous topography of the California coast.



Geostrophic adjustment in a rotating channel, seen from above. The 1 ½ layer density stratification has a thicker upper layer at the left (green), held back by a vertical barrier. The barrier is removed and the green fluid surges along the channel wall as a 'Kelvin wave jet' (red rectangle). But a more simple linear Kelvin wave has propagated ahead, as seen by the movement of the blue dye lines (for example, in the blue rectangle).

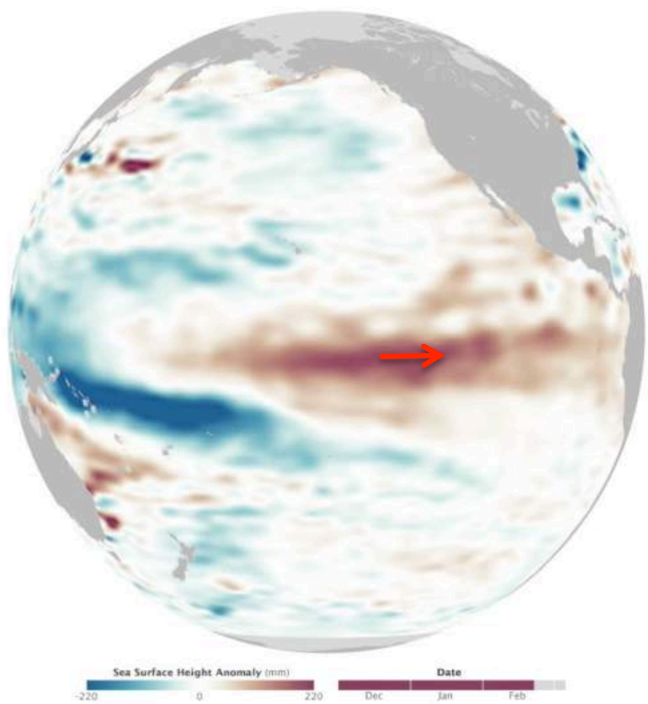
The geostrophic flow has a long life cycle, with instability of the front between green and clear fluid creating a mass of eddies which follow the Kelvin pathway, though are much wider than the initial boundary current, and eventually fill the channel. This later stage involves potential vorticity dynamics and flow instability; Hermann *et al.* J.Fluid Mech. 1989.

Equatorial Kelvin waves. The atmosphere has no vertical walls, but being spherical, it has a Coriolis frequency that varies with latitude. The simulation of a free particle on a sphere showed oscillations near the Equator that relied on this variation. With f changing sign at the Pacific Equator we can imagine a Kelvin wave in the northern hemisphere propagating eastward (that is cyclonically with respect to the North Pacific basin), and another Kelvin wave in the southern hemisphere propagating cyclonically with respect to the South Pacific: that is, also propagating eastward! The two Kelvin waves 'lean on' one another, with the Equator a line of symmetry, forming a very rapid, eastward propagating wave with latitude scale equal to an equatorial version of the Rossby radius. We defined $L_D = Co/f$ in midlatitude, but f varies near the Equator nearly linearly, $f \approx \beta y$. Here suppose we use an average value of f over the distance L_D; so

$$L_D = Co/\beta L_D \quad => \quad L_D = (Co/\beta)^{1/2}.$$

For the non-rotating gravity wave propagation speed we use a 1 ½ layer stratification, $Co^2 = g'h$ where h is the upper layer thickness. For the ocean this gives a Kelvin wavespeed of about 3 m sec^{-1} with a north-south length scale L_D of about 400 km. Equatorial Kelvin waves in the atmosphere have modes with wavespeeds typically 20 to 80 m sec^{-1} and north-south scale of about L_D roughly 6 to 12 degrees of latitude (~200-1300 km).

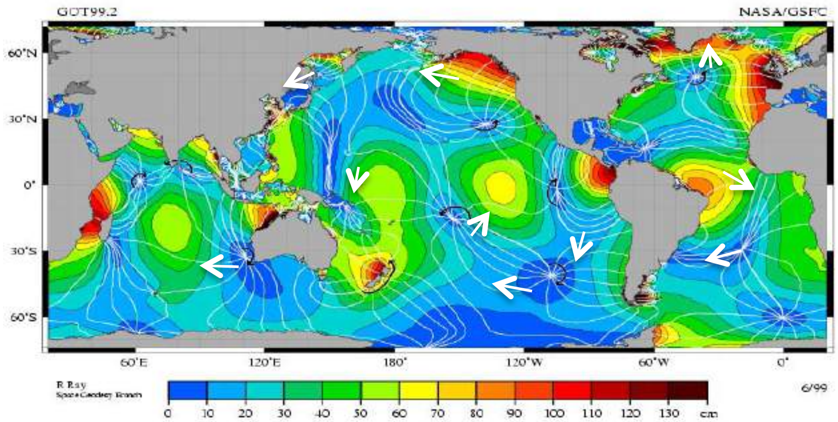
Eastward propagating Kelvin waves and (1/3 as fast) westward propagating Rossby waves make up an equatorial 'oscillator' that is essential to el Niño/Southern Oscillation cycling. The oceanic Kelvin wave can be seen at http://visibleearth.nasa.gov/view.php?id=43105

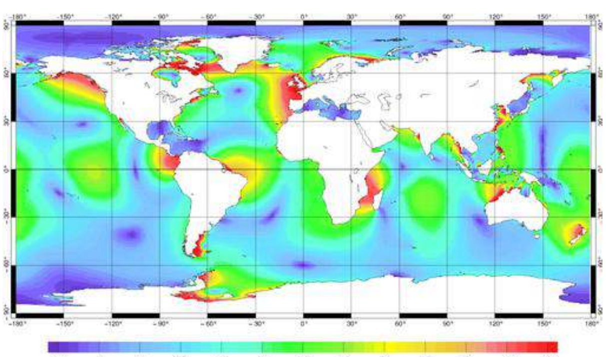


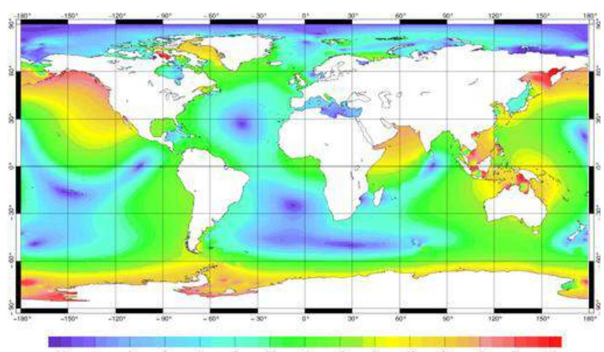
Eastward propagating oceanic Kelvin wave seen with satellite altimetry. el Niño involves a surge of warm water/high sea-surface elevation (SSH) 'released' from the western Pacific warm pool when the easterly (westward) Trade Winds weaken. This is the mirror-image nature of a 1 ½ layer stratification, where the SSH deforms oppositely to the base of the thermocline. NASA Earth Observatory images by Jesse Allen, Kevin Ward, and Robert Simmon. Caption by Rebecca Lindsey, based on interpretation provided by Josh Willis and Bill Patzert, NASA JPL.

The ocean tides are in many regions essentially Kelvin waves. Their amplitude near the coasts is greater than in mid-ocean

tides







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