GFD1 AS509/OC512 Winter Q. 2017 Dargan Frierson Tues 7 Feb 12.30
Lab 5 P.B.Rhines Ekman layers, spin-up and spin-down of circlation

Generation of the energy of the circulation of fluids consumes a lot of our attention. Equally important is the dissipation, damping or removal of the mechanical energy of the circulation which closes the energy cycle. Whereas heating of the fluid is an important generator of circulation, viscous energy dissipation converts KE and PE back to heat energy (that dissipative heating usually too small to feed back on the circulation). Viscous drag also acts as a break on the momentum of the fluid.

We can see the cumulative effect of drag of the atmosphere on the solid Earth by simply measuring the length of the day. Hide and Dickey (1991) show that in northern-hemisphere winter, when the winds are strong and land surface is great, the solid Earth's angular momentum accelerates. The annual cycle is about 1 millesecond in the length of the day. Some of this, roughly ½ , is inviscid 'Bernoulli' pressure drag on mountain slopes, but the rest is turbulent-viscous frictional of the winds on the land beneath.

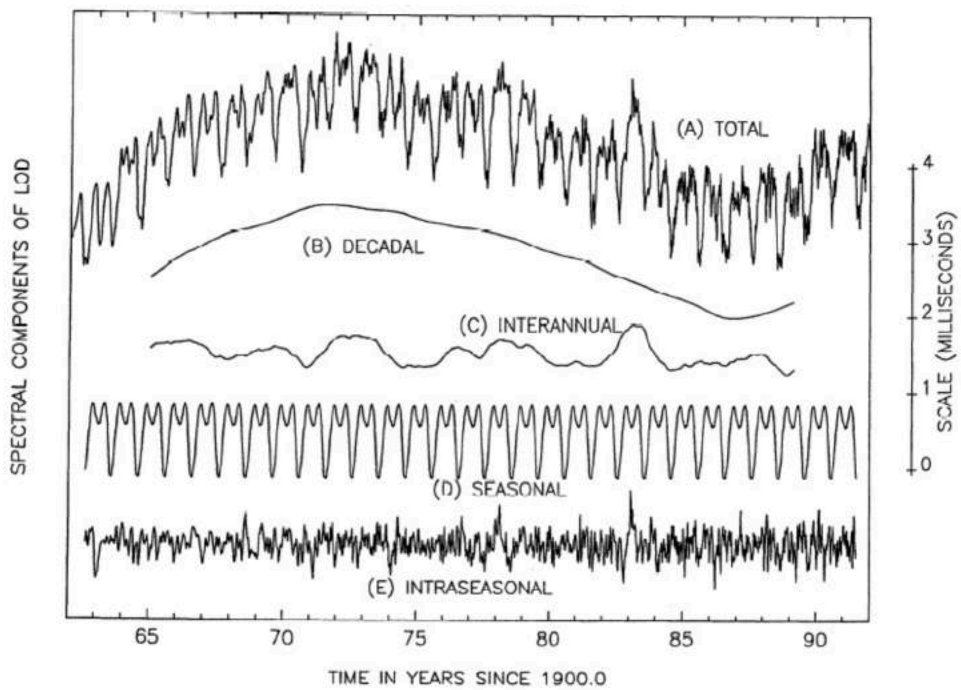


Figure 1. Time series of irregular fluctuations in the length-of-day $\Lambda^*(t)$ (curve A) and its decadal ($\Lambda_\alpha(t)$), interannual ($\Lambda_\beta(t)$), seasonal ($\Lambda_\gamma(t)$), and intraseasonal ($\Lambda_\delta(t)$) components (curves B, C, D, and E respectively) updated from *Hide and Dickey* [1991].

Classical fluid dynamics study often begins with inviscid potential flow. Viscous forces then are considered and these create vorticity in the flow, particularly near a solid boundary or interface with another fluid (as at the sea surface). The connection between vorticity $\vec{\omega}$ and frictional effects can be seen by rewriting the viscous force for an incompressible fluid

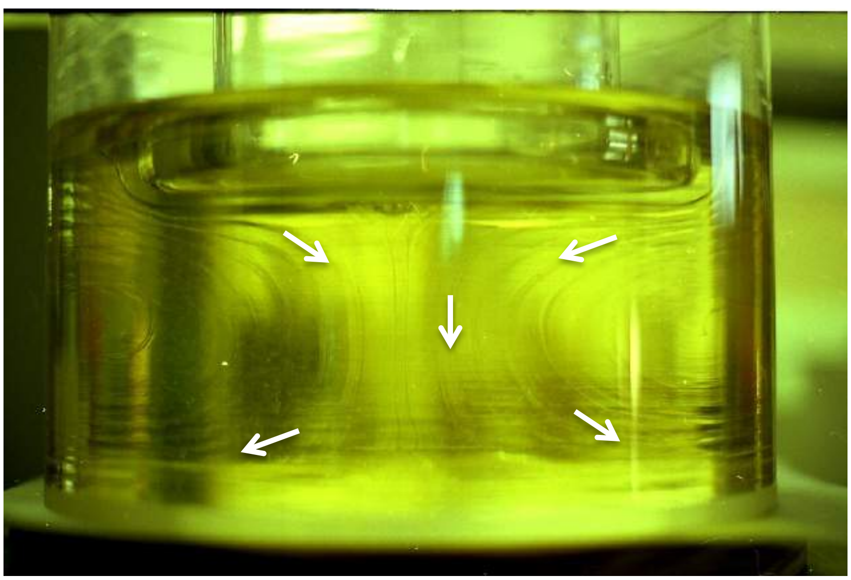
$$\nu\nabla^2\vec{u} = -\nu\nabla\times\vec{\omega};\ \ \vec{\omega}\equiv\nabla\times\vec{u}$$

ν is the kinematic viscosity coefficient[&]. In a rotating fluid, relative vorticity is also created or altered by purely inviscid stretching of planetary vortex lines. These two ideas come together in the 'spin-up' problem, which describes how viscous boundary layers at the base of the ocean or atmosphere either create or destroy the circulation throughout the fluid. Transmission of the effect of near-boundary viscous flow changes the circulation throughout the water column, using purely inviscid vortex stretching. This interaction involves 'overturning circulations', often rather weak velocities which are oriented normal to the rapidly flowing primary, horizontal flow. Thermally driven overturning circulations..Hadley cells... alter the general circulation (typically the zonal flow) in the same way.

The tea-leaf effect. Without needing to consider Coriolis forces we can easily see a prime example of boundary layer friction in a tea-cup. Loose tea leaves, sitting on the bottom, strangely converge on the center of the cup if the tea is stirred to form a swirling, azimuthal ('zonal') flow. We might have expected our stirring to mix the leaves out turbulently but they 'unmix'. The idea is that the spinning liquid has a radial pressure gradient $\partial p/\partial r$, the force that balances the acceleration of fluid parcels following circular paths: $1/\rho\ \partial p/\partial r = u^2/r$ where u is the azimuthal velocity. In a thin layer near the bottom boundary, viscous forces slow the fluid and yet the pressure gradient is still virtually the same (because the vertical velocity near a horizontal boundary is very small, so the vertical momentum equation has little to balance a vertical pressure gradient, $\partial p/\partial z$). This pressure gradient pushes fluid and teal leaves toward the center in that boundary layer.

To see this in action we exaggerated the viscous effect by using glycerine rather than water. Glycerine is a water-soluble oil with density similar to water but viscosity about 1000 times as great.

[&] The full vector identity is $\nabla^2 \vec{u} = -\nabla \times \vec{\omega} + \nabla(\nabla \bullet \vec{u})$; $\quad \vec{\omega} \equiv \nabla \times \vec{u}$



A cylinder of glycerin is rotated rapidly, with an upper boundary that is not rotating at all (stationary in the lab frame of reference). Thus the top and bottom boundaries are exerting opposite viscous forces on the fluid, and it ends up rotating at a rate roughly halfway between. Again, the inward pressure force $\rho u^2/r$ balances $\partial p/\partial r$ at mid-depth but near the top the azimuthal (zonal-) velocity is reduced by viscous drag, so the pressure force is free to push fluid inward (white arrows in the figure). The bottom boundary rotates more rapidly than the interior fluid, so its viscous force flings fluid outward in the bottom boundary layer. Using water, the Ekman boundary layer thickness $\delta = (\nu/f)^{1/2}$ is about 1 mm. ... extremely thin. With ν about 1000 times greater, δ is roughly 3.3 cm and hence very visible...occupying nearly half the depth of the glycerine.

The details of the Ekman spiral (the changing speed and direction of the horizontal velocity as one approaches the boundary) are intricate, but the net effect, the net transport of fluid in the Ekman layer $\vec{M}$ is simple : $\vec{\tau} = f\hat{z}\times\vec{M}$ $\quad \vec{M} \equiv \rho\int_0^\infty (\vec{u}-\vec{U})dz$

Here $\hat{z}$ is a vertical unit vector, f is Coriolis frequency and $\vec{u}-\vec{U}$ is the difference between the velocity in the boundary layer and the geostrophic velocity in the interior, $\vec{U}$. The viscous stress on the boundary is

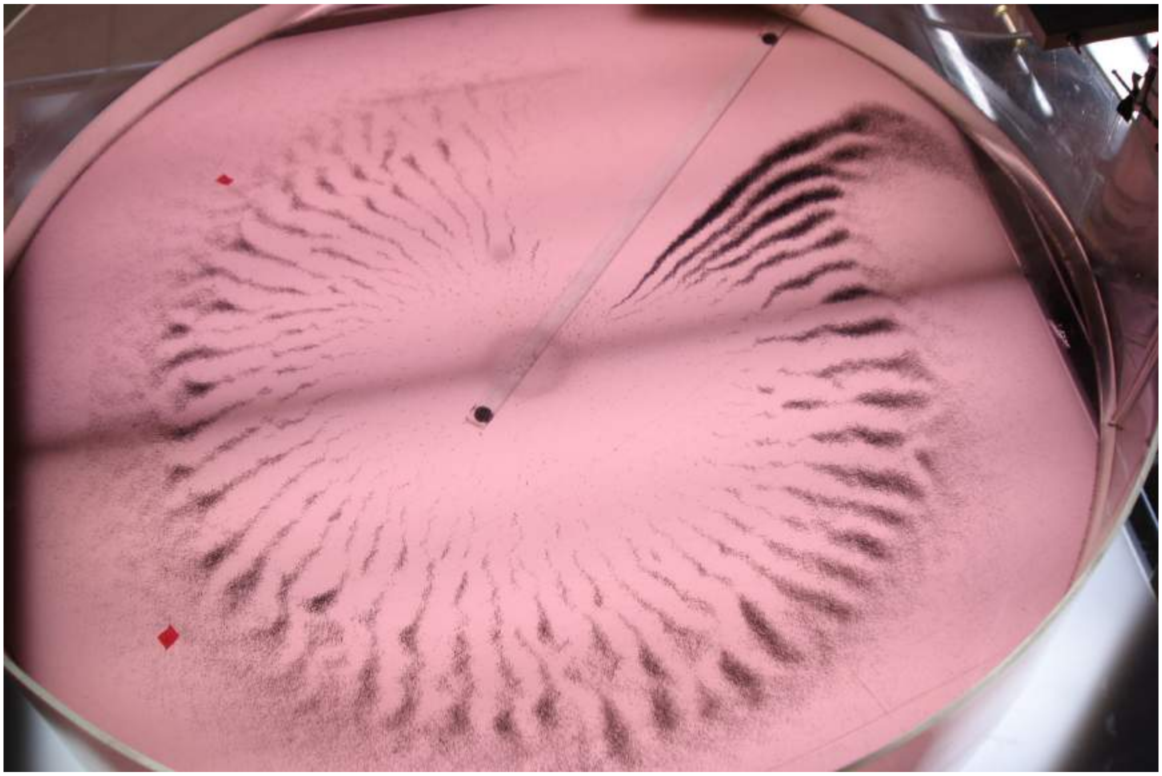
$$\vec{\tau} = \rho\nu\frac{\partial\vec{u}}{\partial z} ,$$

evaluated at the boundary. Of course oceans and atmospheres have turbulent boundary layers much thicker than 1 mm. The power of the theory is that we know the volume transport of the Ekman layer as a function of boundary stress, even for a turbulent flow if it is relatively uniform statistically and without strong stable or convective stratification. This horizontal transport is perpendicular to the stress. Typically we observe or estimate stress as a function of the interior geostrophic flow velocity, or in the upper ocean, the wind-stress. A typical estimate then for δ is

$$\delta \sim u^*/f$$

where u* is just a way of writing the stress: $|\vec{\tau}| \equiv \rho u^{*2}$. For the ocean this gives Ekman layer thicknesses of order 5 -10 m and for the atmosphere, several hundred m.

Cooling of the water surface in the large rotating cylinder, by evaporation, causes small convection cells, which spin like as miniature tornadoes. The cyclonic surface swirling is visible, the 'tree-trunks' show where dye is pulled downward from the surface, and at the base of each vortex core, the blue dye shows the Ekman spiral: the change in direction and speed of the horizontal velocity in the lowest few mm. of the water column. For cyclonic vortices we expect the Ekman transport at the bottom to be inward.



Thin Ekman boundary layers are unstable, and turbulent rolls and eddies thicken them.

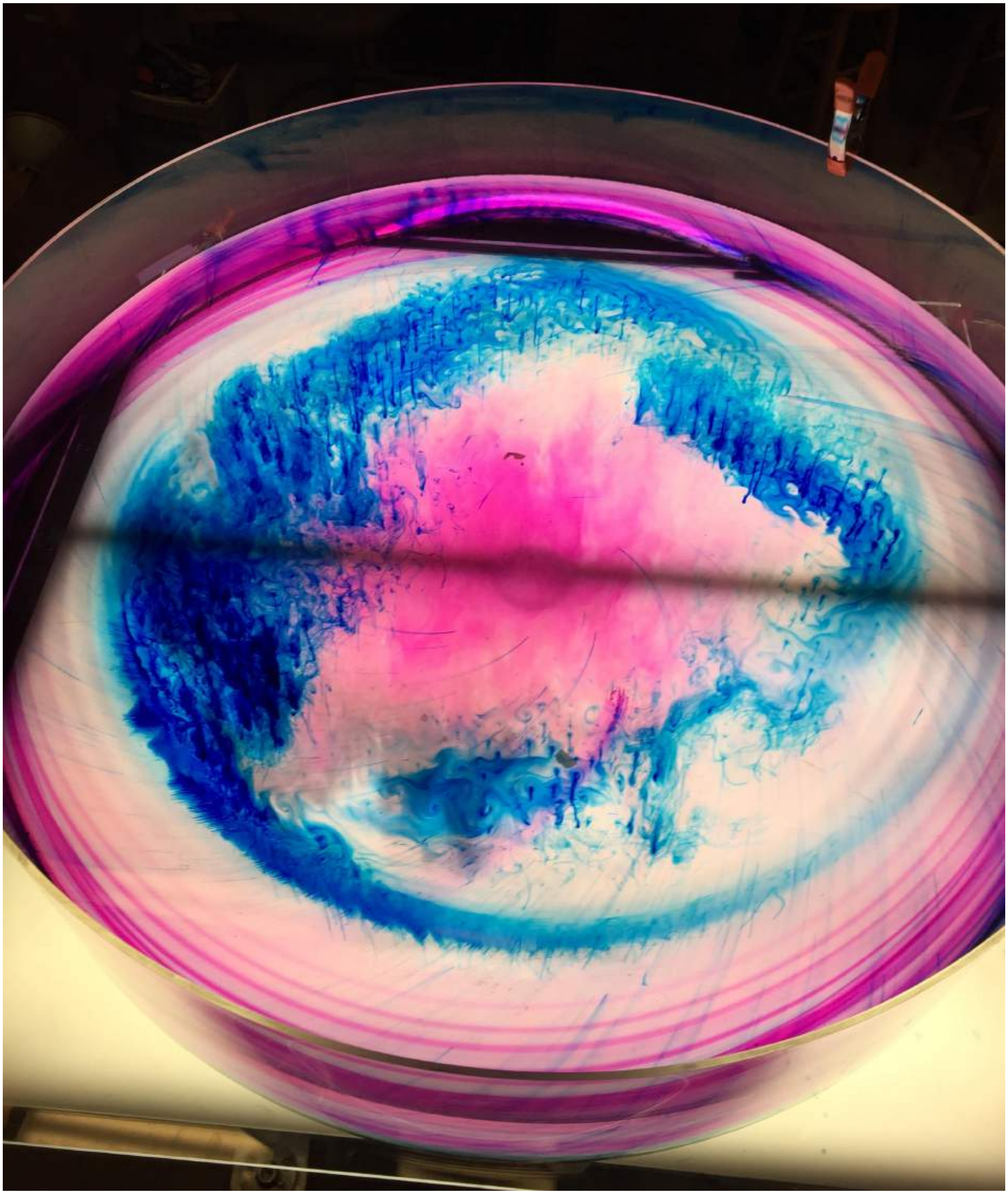
Spin-up and spin-down. The overturning circulation visible in the glycerine experiment occurs also with water or air, but with small viscosity the radial flow is confined to the very thin Ekman layers. The vertical velocity they induce stretches or squeezes the interior fluid, changing its relative vorticity. The theoretical detail establishes how the Ekman transport forces the vertical w-velocity of the interior, and the interior responds according to the vorticity equation

$$\frac{D\zeta}{Dt} = (f+\zeta)\frac{\partial w}{\partial z} \quad \Rightarrow \quad \frac{\partial \zeta}{\partial t} = f\frac{\partial w}{\partial z} \qquad \zeta \equiv v_x - u_y$$

The second expression is for low Rossby number, nearly geostrophic flow. The viscous boundary layer communicates with the interior when it is horizontally divergent: the divergence $u_x + v_y = -w_z$ forces w which matches the w of the interior. This turns out to mean that Ekman pumping of the interior fluid is proportional to the difference in vorticity of the boundary and fluid.

In the lab we spun the large cylinder up and down, showing the Ekman transport with purple dye crystals on the bottom. The theory based on the equation above says that the flow spins up to the speed of the boundary in a time ~ $H/\delta f$ where h is total depth. This means that only a small fraction (~ δ/h) of the fluid has to pass through the boundary layer in order to achieve spin-up. Hence the spin-up time is much faster than the simple viscous diffusion time for a layer this thick (~ h^2/ν). Inserting $\delta = (\nu/f)^{1/2}$, we find the ratio of the Ekman spin-up time to the simple momentum diffusion time is thus just δ/f << 1.

Viscous spin-down (combined with pressure drag on mountains) balances the momentum forcing of the general circulation, and also combines with interior turbulent KE dissipation to balance the energy-forcing of the circulation. Spin-down of tropospheric weather systems occurs over a few days, owing to the great thickness of the planetary boundary layer. The ocean's viscous spin-down time is more like 100 days. However density stratification 'insulates' the interior baroclinic circulation of the fluid from Ekman spin-down to some extent, and we find upper ocean eddies with lifetimes of several years!



Spin up of homogeneous water on the large rotating table. The outward Ekman flow is visible in faint blue lines. Permanganate crystals produce a deep purple layer which migrated to the outer wall, climbing up it in rings. (I don't know what instability caused the rings to develop). The dark blue dye was inserted at the surface near the outer boundary but it has now been squeezed inward by the meridional overturning circulation, to achieve the final spin up, as the fluid comes to the same rotation rate as its solid boundaries. As usual, convective tornadoes are everywhere.