ATM S 509/OCEAN 512
Frierson
HW set 3 (due 2-24-17)

1) Thermal wind for angular momentum conserving winds. Use the steady-state, equatorial beta-plane ($f = \beta y$), anelastic equations of motion, also assuming zonal symmetry (so all x-derivatives can be assumed to be zero). Take $\theta_0 = 300K$, $H = 10km$, and the appropriate equatorial value of beta.

   i) Confirm that the governing equations are

$$vu_y + wu_z - \beta yv = 0$$
$$vv_y + wv_z + \beta yu = -\phi_y$$
$$\phi_z = b = \frac{g\delta\theta}{\theta_0}$$

   ii) Use the x-momentum equation to derive an equation for the zonal winds in the upper troposphere, which start at zero as they rise at the equator and increase away from this. Assume the vertical advection term is small. This is the "angular momentum conserving wind" for the beta-plane equations.

   iii) Use the y-momentum equation to obtain an expression for the anelastic pressure ϕ, assuming again that vertical advection is small, and that the meridional velocity is much smaller than the square root of the pressure variable ϕ.

   iv) Assume that the surface zonal winds are negligible near the surface to calculate the vertically averaged buoyancy perturbation and potential temperature difference from the equator. What is this potential temperature difference at 10, 20, 30 degrees? (Remember $dy = ad\theta$ where θ is the latitude measured in radians) If winds are sub-angular momentum conserving (i.e., if the upper tropospheric winds are less in magnitude than the value you calculated in part ii), what does that mean for the meridional gradient in potential temperature across the tropics?

2) Vertical velocities and stretching in the shallow water equations. Show that in the shallow water equations with no bottom topography ($\eta_b = 0$), the vertical velocities satisfy $w = \frac{z}{\eta}\frac{D\eta}{Dt}$. Show therefore that a parcel stretches uniformly, i.e., a parcel halfway up the column remains halfway up the column.

3) Streching of absolute vorticity in the shallow water equations. Consider a shallow water fluid with planetary vorticity *f*, initially with cyclonic relative vorticity $\zeta = 0.1f$ (note this situation has Rossby number 0.1, similar to atmospheric

and oceanic conditions). Stretch this fluid to twice its depth. What is the relative vorticity after stretching? Squash it to have half its original depth. What is the relative vorticity now? Do the same two calculations for initial anticyclonic vorticity $\zeta = -0.1f$.