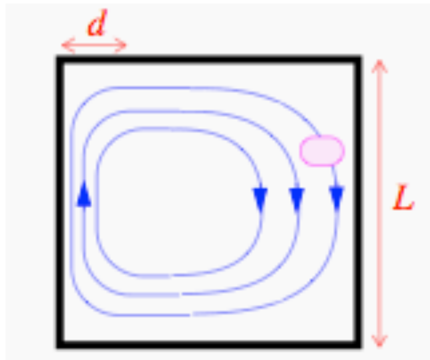
ATM S 509/OCEAN 512
Frierson
HW set 1 (due at beginning of class on 1-20-17)

1) Assume an ocean basin-like flow with an intensified western boundary current, like the Gulf Stream or Kuroshio. The equatorward flow is spread roughly uniformly over most of the domain, but the poleward flow is concentrated in a layer of width $d \ll L$. Using scaling arguments, estimate the time for the blob of tracer in the diagram to be diffused across the basin. Hint: compare advective and diffusive times in the interior and the western boundary current.



2) Static stability of the tropics with global warming: The tropical tropospheric temperature profile is determined by moist convection. Assume the convective parcels start at the surface where the pressure is 1000 hPa, the temperature is 300 K, and the relative humidity is 80%.
   a. What are the potential temperature and the equivalent potential temperature of this surface air?
   b. Assume convection occurs up to p_t=125 hPa and z_t=15 km, where all of the moisture has condensed out, but the equivalent potential temperature is the same. What is the potential temperature at this level? What is the temperature at this level?
   c. Estimate the average buoyancy frequency of the profile. You can use the discretized formula

$$N^2 = \frac{g}{0.5(\theta_t + \theta_s)} \frac{\theta_t - \theta_s}{z_t}$$

   d. If the surface temperature rises by 4 K with global warming, and the surface relative humidity stays the same, how do the equivalent potential temperature and the potential temperature at 125 hPa change? How does the buoyancy frequency change (calculating up to 125 hPa as in part c)? How much more does the temperature at 125 hPa increase compared with the surface warming?

3) Derive the following forms of the thermodynamic equation for an ideal gas:

$$c_V \frac{DT}{Dt} = -RT\ \nabla \cdot v + Q$$

$$\frac{Dp}{Dt} = -\gamma p \nabla \cdot v + \frac{R}{c_V} \rho\, Q$$

$$\frac{DT}{Dt} = \kappa \frac{T\omega}{p} + \frac{Q}{c_p} \quad \text{where } \omega = \frac{Dp}{Dt} = \text{pressure velocity}$$

4) Let $(u,v) = (By,\ 0)$ and let the initial condition be $A = A_0 \cos(kx)$. Solve the 2-D advection-diffusion equation exactly for this flow. Comment on how the shear changes the typical time scale for diffusion.

Hint: Look for solutions of the form

$$A = exp\left(ik[x - u(y)\ t]\right)\ T(t)$$