ATM S 509/OCEAN 512

Frierson

Final Exam (due 3-17-17 by 4 PM)

Open book, open notes, work independently, no collaboration of any kind.

Choose any 3 of these 4 problems.

1) Consider an ideal gas, in both geostrophic and hydrostatic balance. We'll be working in a slightly modified version of pressure coordinates in this problem.

   a. Show that hydrostatic balance can be written as $\frac{\partial \Phi}{\partial \Pi} = -\theta$, with $\Phi = gz$ = the geopotential, $\Pi = c_p \frac{T}{\theta}$ = the Exner function, and θ = the potential temperature.

   b. Derive a thermal wind equation relating $\frac{\partial u}{\partial \Pi}$ to $\frac{\partial \theta}{\partial y}$. Provide a physical interpretation of this relation. Show that within layers of constant potential temperature, *u* is independent of pressure.

   c. Consider a two layer fluid with constant potential temperature θ_1 in the top layer and θ_2 in the bottom layer. Derive expressions for the geopotential gradient in each layer as a function of the surface Exner function and the Exner function at the interface.

   d. Derive an equation for the velocity difference between the two layers, as a function of the slope of the interface and the potential temperature difference $\theta_1 - \theta_2$.

   e. Suppose the pressure of the layer interface slopes up from 700 *hPa* to 400 *hPa* between 30 and 60 degrees latitude. Calculate the wind shear necessary to support this arrangement if $\theta_1 - \theta_2 = 30\ K$.

2) Consider a shallow water flow on an f-plane along the continental shelf of the ocean. Assume that the shelf slopes downward in the westward direction, so $H(x) = H_0 + \alpha x$.

   a. What is the background potential vorticity for this problem?

   b. Construct the linearized PV equation for this system about a state of rest.

c. Calculate the dispersion relation for topographic Rossby waves in this system. What direction do Rossby waves propagate? Draw a diagram indicating the propagation mechanism of these waves.
d. What is the maximum frequency of these waves? How large of a slope is required to generate phase speeds similar to that of Rossby waves on a beta-plane at 45 degrees?

3) Consider the Boussinesq equations on a beta-plane (so $f = f_0 + \beta y$).
   a. What is the horizontal divergence of the geostrophic velocity in this system?
   b. Consider the Ekman equations in this system, in the presence of a cyclonic wind stress at the surface equal to $\tau = -Ae^{-r^2/\lambda^2}(y\hat{i} - x\hat{j})$ with $r^2 = x^2 + y^2$. Calculate the Ekman mass transport and "Ekman pumping" ageostrophic vertical velocity at a depth where the stress has essentially disappeared. Plot the Ekman pumping for different values of lambda.

      Note: you can keep everything written in terms of tau for this problem, i.e., you don't need to model the stress as a diffusion with $\frac{1}{\rho}\frac{\partial \tau}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2}$.

4) You're a member of the elite UW SWAT (shallow water attribution team). Get to work!
   a. You're told that a Poincaré wave is coming through a linear shallow water system on an f-plane, but you're only allowed to see a glimpse of the fields at one instantaneous time slice. How can you figure out which direction the wave is propagating?
   b. How can you tell what the wavelength is as compared to the Rossby radius? (You can watch a few cycles of the wave if you need to for this part)
   c. How can you determine what will be left behind when the wave passes through?