GFD I

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Summary so far…

- Derived the **shallow water equations**
  - Simple system with ***no vertical structure***
    - Velocity independent of depth within the layer
  - Three equations for *u, v, h/eta*
  - For reference, full SWEs are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -g\frac{\partial h}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\frac{\partial h}{\partial y}$$

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0$$

Summary so far…

- Waves in the SWEs:
  - Showed that "**The Wave Equation**" from applied math can be derived exactly from the SWEs
    - With c = sqrt(g H)
  - Wave speed is independent of scale:
    - All Fourier modes propagate at same speed
    - Not true for other waves we'll consider in the class!
- Still the subtlety of what goes east and what goes west…

Summary so far…

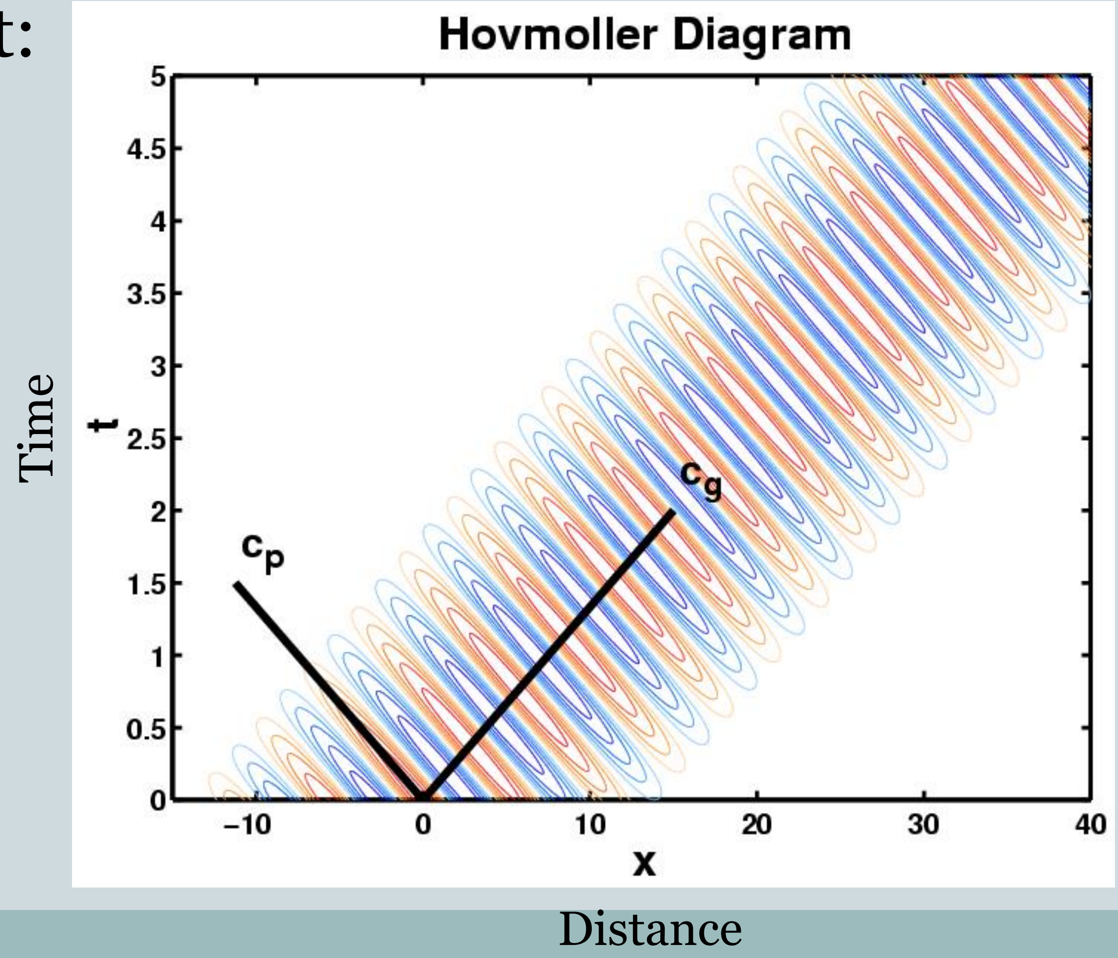
- Waves in the SWEs:
  - "Modes": structures that exactly keep their shape
    - Velocity proportional to height in east-moving mode
    - Velocity opposite to height in west-moving mode
  - Projection onto different modes determines what goes east & what goes west

Summary so far…

- Rotating shallow water waves
  - Also called "Inertia-gravity waves"
    - Propagation characteristics are strongly a function of wavenumber (i.e., these waves are **dispersive**)
  - First appearance of the famous "Rossby Radius of Deformation"
    - For length scale << Rossby radius, rotation is not felt
      - Dispersion relation is same as for non-rotating waves
    - For length scale >> Rossby radius, rotation is dominant
      - These are "inertial oscillations" or "constant angular momentum oscillations"
      - Group velocity is zero in this case

Dispersion

- Dispersion is when phase velocity & group velocity are different:



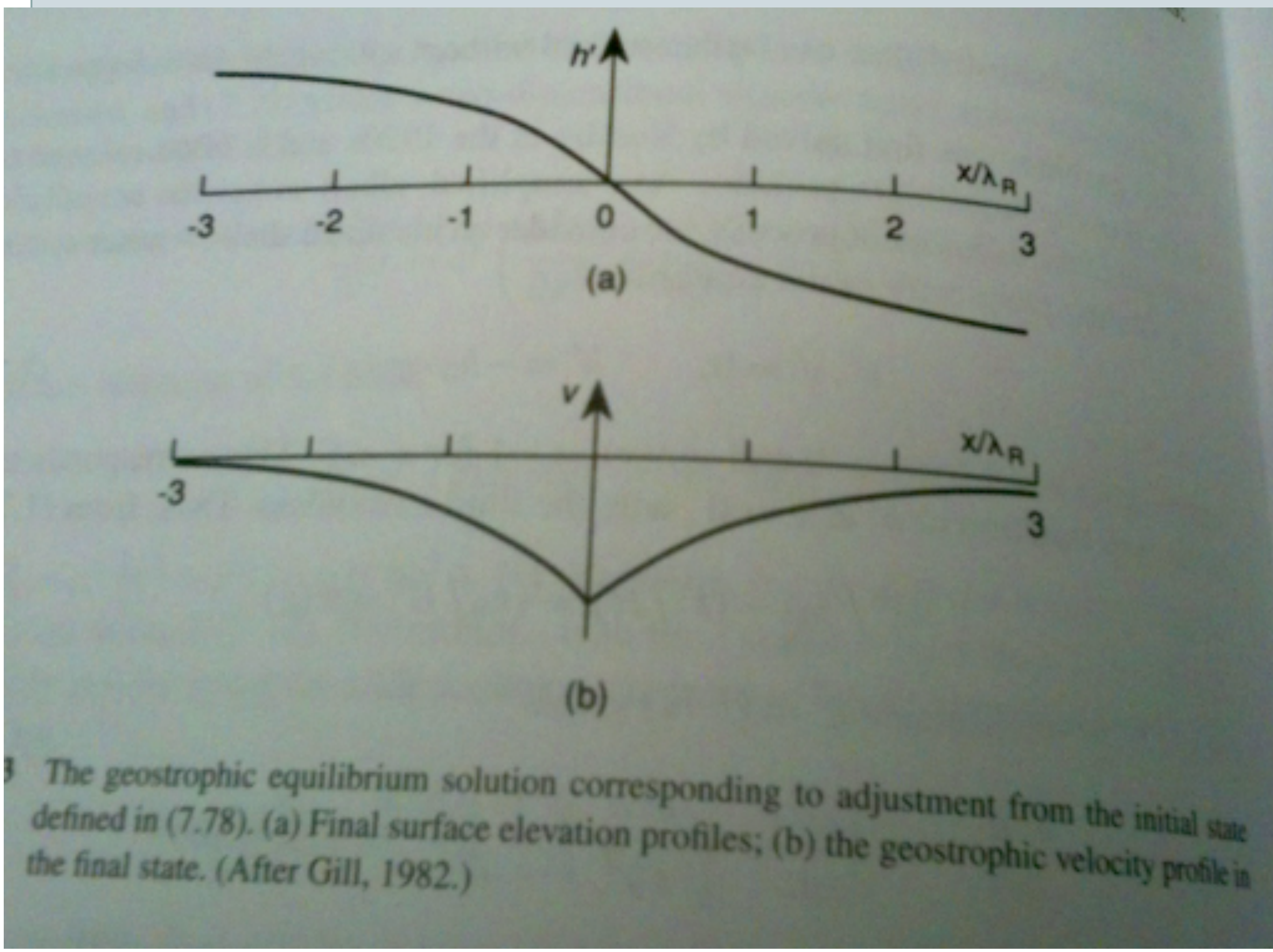
Next: Geostrophic Adjustment

- The “dambreak” problem


THE DAMBREAK PROBLEM
쓰나미도 휩쓸지 못한 그들의 이야기가 시작된다
BETTER HOPE YOU'RE A ROSSBY RADIUS AWAY
해운대
설경구 하지원 박중훈 엄정화 이민기 강예원 김인권 | 윤제균 감독작품 | 7월 대개봉
CGV 채널CGV가 한국영화를 응원합니다

Dambreak Problem Steady State

- Steady state height and v fields:



The geostrophic equilibrium solution corresponding to adjustment from the initial state defined in (7.78). (a) Final surface elevation profiles; (b) the geostrophic velocity profile in the final state. (After Gill, 1982.)

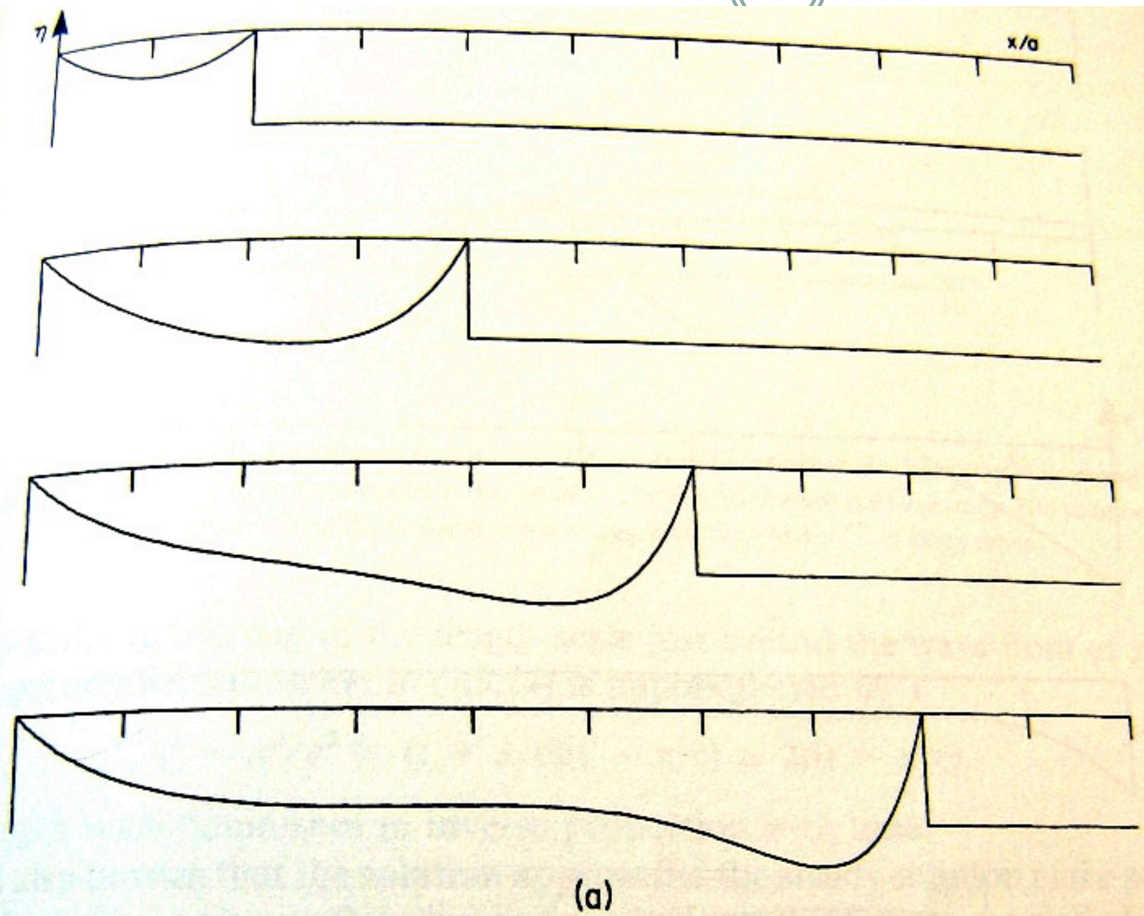
Geostrophic Adjustment: Summary

- Dambreak problem.
  - Goal: steady state *u*, *v*, *h* fields.
- Regular equations are degenerate (need extra constraint to solve).
- PV equation provides this: PV stays exactly the same at all points!
- Steady state solution:
  - *u' = 0.*
  - *h' = unchanged (far away compared to Rossby radius).*
  - *h' ~ approx'ly linear (well inside a Rossby radius)*
  - *v' = in geostrophic balance with h'*
  - *v' = forms a jet in negative direction*

Geostrophic Adjustment

- We calculated the steady state solution. What about the transient response?
- See Gill for full time-dependent solution.
- Let's examine qualitatively though.
- Adjustment is accomplished by inertia-gravity waves!
  - Short waves are essentially nonrotating
  - Long waves have slower group velocities and are more strongly influenced by Coriolis.

Transient Geostrophic Adjustment: Height Field



(a)

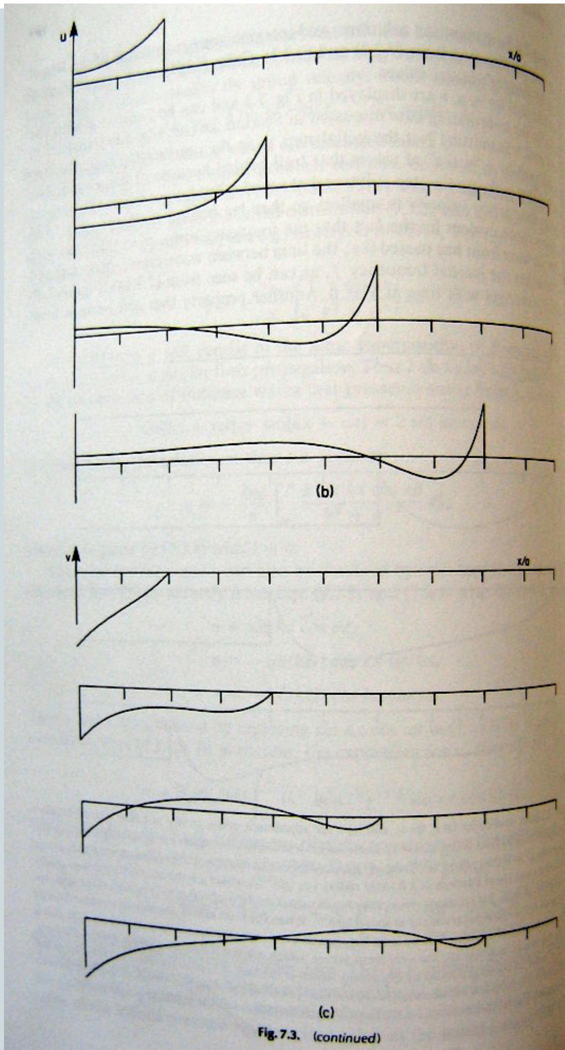
3. Transient profiles for (a) η, (b) u, and (c) v for adjustment under gravity of a fluid with an ini
discontinuity in level of $2\eta_0$ at $x = 0$. The solution is shown in the region $x > 0$, where the surfa
pressed, at time intervals of $2f^{-1}$, where f is twice the rate of rotation of the system about a vertic
on the x axis are at intervals of a Rossby radius, i.e., $(gH)^{1/2}/f$, where g is the acceleration due to

Height field at 4 consecutive times (just the positive x-axis is plotted – the part that was initially negative).

Note the *front* and small scale behavior moves at the *gravity wave speed.*

Longer wavelength stuff follows along more slowly.

Transient Geostrophic Adjustment: *u* and *v* Fields



- **Zonal winds at the same 4 times and locations as previous slide**.
- Zonal winds become positive initially due to initial pressure grad force.
- Oscillations due to inertial oscillation-type behavior.
- Goes to zero eventually.

- **Meridional winds at same 4 times.**
- Note *no front* in this field.
- Jet forms immediately (responding to positive zonal wind anomalies).
- Positive meridional wind anomalies also occur due to inertial oscillation-type behavior.