Potential Temperature

$$dh = C_v\,dT + p\,d\alpha \qquad \alpha = \frac{1}{\rho}$$

adiabatic $\rightarrow dh = 0$

$$C_v\,dT + p\,d\alpha = 0$$

$$d(p\alpha) = p\,d\alpha + \alpha\,dp$$

$$\rightarrow \quad C_v\,dT + d(p\alpha) - \alpha\,dp = 0$$

use $\quad C_p = C_v + R$ and $p = \rho RT$
$\qquad\qquad\qquad\qquad\qquad p\alpha = RT$

$$C_v\,dT + R(dT) - \alpha\,dp = 0$$

$$(C_v + R)dT - \alpha\,dp = 0$$

$$C_p\,dT - \alpha\,dp = 0$$

$$\rightarrow \quad \frac{C_p}{R}\frac{dT}{T} = \frac{dp}{p} = 0$$ using $p = \rho RT$

integrating from $p_0 (1000) \rightarrow p$
$\qquad \Theta = T_0 (1000) \rightarrow T$

$$\frac{C_p}{R}\int_{\Theta}^{T}\frac{dT}{T} = \int_{p_0}^{p}\frac{dp}{p}$$

$$\rightarrow \frac{c_p}{R} \ln \frac{T}{\theta} = \ln \left(\frac{p}{p_0} \right)$$

$$\rightarrow \left(\frac{T}{\theta} \right)^{c_p/R} = \frac{p}{p_0}$$

$$\rightarrow \theta = T \left(\frac{p_0}{p} \right)^{R/c_p}$$

poissons eq for

potential temp. → temp at 1000 mb
when you bring an air parcel
down dry adiabatically