A Technique for Computing Vertical Transports by Precipitating Cumuli

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ABSTRACT

A method is described for calculating cumulus-scale vertical transports of mass, sensible heat and horizontal momentum from detailed measurements of precipitation. The basic premises are that the amount of lifting in a cumulus cell is related to the precipitation which it produces and that the temperature excess and entrainment are reflected in its vertical development. The amount of cellular precipitation may be obtained from quantitative radar data or high-resolution rain gauge records, the cell depths from radar and radiosonde data.

The mass of air transported upward within the cells is computed from the conservation of water given the amount of cellular precipitation. A relationship between measured cellular rain and condensate within cumulus cells is based on available empirical information. Entrainment rate and the shape of the mass transport curve as a function of height are specified in a manner consistent with physical considerations, experimental evidence, and one-dimensional dynamic models of cumulus cells. For the transport of sensible heat the temperature excess within the cells is computed from conservation equations for water and vertical momentum. For the transport of horizontal momentum, the difference in horizontal wind speed is calculated from the conservation of horizontal momentum and the drag force exerted by the environment on the air in the updrafts.

Uncertainties in the computed transports are obtained by estimating limiting values for the assumptions involved.

1. Introduction

Widespread cumulus activity is typical of tropical regions, where it plays an important role in the development and maintenance of larger scale circulations and in the balance of energy and momentum budgets (e.g., Riehl and Malkus, 1958; Charney and Eliassen, 1964). Cumulus-scale precipitation cells occur very frequently in mid-latitude storms as well, as reported by Elliott and Hovind (1964), Browning and Harrold (1969), and Austin and Houze (1972). They are not restricted to summertime showers and thunderstorms but are also common features within most larger scale precipitation systems. The frequency and intensity of precipitating cells suggest that cumulus convection may contribute significantly to vertical transports of such quantities as heat and momentum in temperate as well as in tropical zones.

Estimates of cumulus transports have usually been based on considerations of energy and momentum budgets with the actual measurements of meteorological quantities made on a larger scale (e.g., Palmén and Newton, 1969; Starr *et al.*, 1970; Bunker, 1971; Yanai *et al.*, 1973). More direct computations of the transports, based on observations of the individual cumuli, are generally not feasible because most meteorological quantities cannot be measured with resolution comparable to the dimensions of cumulus cells, except in a limited way as along the flight path of an instrumented aircraft. Sikdar *et al.* (1970) and Sikdar and Suomi (1971) have noted that in convective systems, which are either relatively isolated or exceptionally severe, growth of the area of cirrus outflow from the top can be observed on satellite photographs. They developed a technique for estimating vertical mass flux and released latent heat from such measurements.

Precipitation rate is a quantity which can be depicted in detail over fairly extensive areas or long periods of time with suitably instrumented radars or rain gauges; and we submit that from measurements of precipitation much can be deduced regarding the intensity and effects of cumulus activity. In this paper an effort is made to formulate useful relationships between cumulus-scale vertical transports and observable features of rainfall patterns. Basic premises are that the amount of rain produced in cumulus cells is related to the vertical mass transport within them, and that the depth of the cells is indicative of their temperature excess and entrainment rate. Estimates made by the proposed technique will be rather crude, because assumptions are based on sparse empirical data and highly simplified dynamic cell models. Also, the total transports by cumulus convection will be underestimated because nonprecipitating cumuli are not included and effects associated with

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downdrafts in precipitating cells cannot be fully accounted for with present empirical knowledge. Nevertheless, computations of the type proposed should be useful because they can provide a means of making at least order-of-magnitude comparisons between transports deduced from direct observations of cumulus activity and those derived solely from observed effects on larger scale parameters. This technique should be especially useful at temperate latitudes where a considerable amount of high-resolution rainfall data have been collected and where the study of cumulus transports has been comparatively neglected.

2. Definition of terms

In this paper the term *cell* is used in two ways. It refers to a cumulus-scale system of updraft and downdraft where condensate is produced, some of which falls as precipitation. In the context of precipitation patterns, small areas of relatively intense rain with dimensions similar to those of buoyant updrafts are also called cells.

The term *storm* designates an area of precipitation or a continuous period of rain at any point. The assumed structure of a storm is based on descriptions by Austin and Houze (1972), and is illustrated in Fig. 1. The storm contains cumulus-scale cells which are generally located within mesoscale areas of lighter rain. The mesoscale areas may in turn be located within synoptic-scale regions of still less intense precipitation. In many storms mesoscale precipitation areas are of two sizes and intensities with "small" ones ($\sim 10^2$ km^2) situated inside of "large" ones ($\sim 10^3$ km^2). In Fig. 1, conditions in cumulus-scale, mesoscale, and synoptic-scale precipitation areas, and outside of the rain areas are indicated with subscripts c, m, s and o, respectively. Subscript e refers to air entrained into the cells; it is used to denote quantities which are assumed to have in the mesoscale and synoptic-scale rain areas the same value that they have in the larger scale environment. Although the schematic diagram in Fig. 1 shows only one mesoscale area containing a single cell, it should be noted that a storm may contain several areas of each subsynoptic scale and the rainfall amounts for each are cumulative totals.

The *environment* refers to the air outside of the cells. Distinction must be made, however, between the *general environment*, which presumably is described by soundings, and the small mesoscale precipitation areas which comprise the *immediate environment* of the cells. It is assumed that in the mesoscale precipitation areas the temperature and horizontal winds are the same as those at the same heights in the general environment, but the air is saturated.

3. Formulas for computing the transports

Symbols for the various quantities which enter into the computations are as follows:

R_c	amount of rain in any given storm which falls in precipitation cells as defined above
C	mass of water condensed in cumulus updrafts in a given storm
M_c	mass of air which rises vertically through level z within the cumulus cells
M_{cd}	mass of air transported downward in cellular downdrafts
$(dM_c)_\epsilon$	entrained mass of air entering the updrafts from their environment between z and $z+dz$
$(dM_c)_\delta$	detrained mass of air entering the environment from the updrafts in the layer z to $z+dz$
E	entrainment rate $[=M_c^{-1}(dM_c/dz)_\epsilon]$
ξ_c, ξ_{cd}, ξ_e	value of arbitrary quantity ξ in cellular updrafts, downdrafts, and immediate environment of cells, respectively
q_c, q_e	mixing ratio of water vapor in the cellular updrafts and in their immediate environment
T_c, T_e	temperature in the cellular updraft and in their environment
u_c, u_e	westerly wind components inside and outside the updrafts
z_B, z_T	bottom and top respectively of the layer containing the cells
z_P	top of mesoscale and synoptic-scale precipitation
τ_{sh}	vertical transport of sensible heat
τ_{am}	vertical transport of angular momentum.

With the exception of the rainfall amounts, total condensation and specific heights, the quantities listed above are functions of height. It is assumed that at any given level all of the air rising in the cells has the same characteristics. Physically, this assumption may be interpreted as implying complete and instantaneous mixing within the updrafts, or simply that the values used for the quantities represent averages over the cells at each level.

The net vertical transport of any quantity ξ through level z accomplished by the cellular lifting is

$$T_\xi = M_c(\xi_c - \xi_e) + M_{cd}(\xi_e - \xi_{cd}), \qquad (1)$$

where it is assumed that at any given level the air rising in the cells, M_c, with characteristic ξ_c, is balanced by a mass of air $(M_c - M_{cd})$ subsiding outside the cells with characteristic ξ_e, and a mass M_{cd} descending in the downdraft with characteristic ξ_{cd}. In our calculations the effect of the downdraft is not computed explicitly as indicated by the second term in (1); it may be taken into account by applying an estimated correction to the computed value of the first term (see Section 9). The transports of sensible heat and angular momentum which are calculated for a storm, and may be later cor-

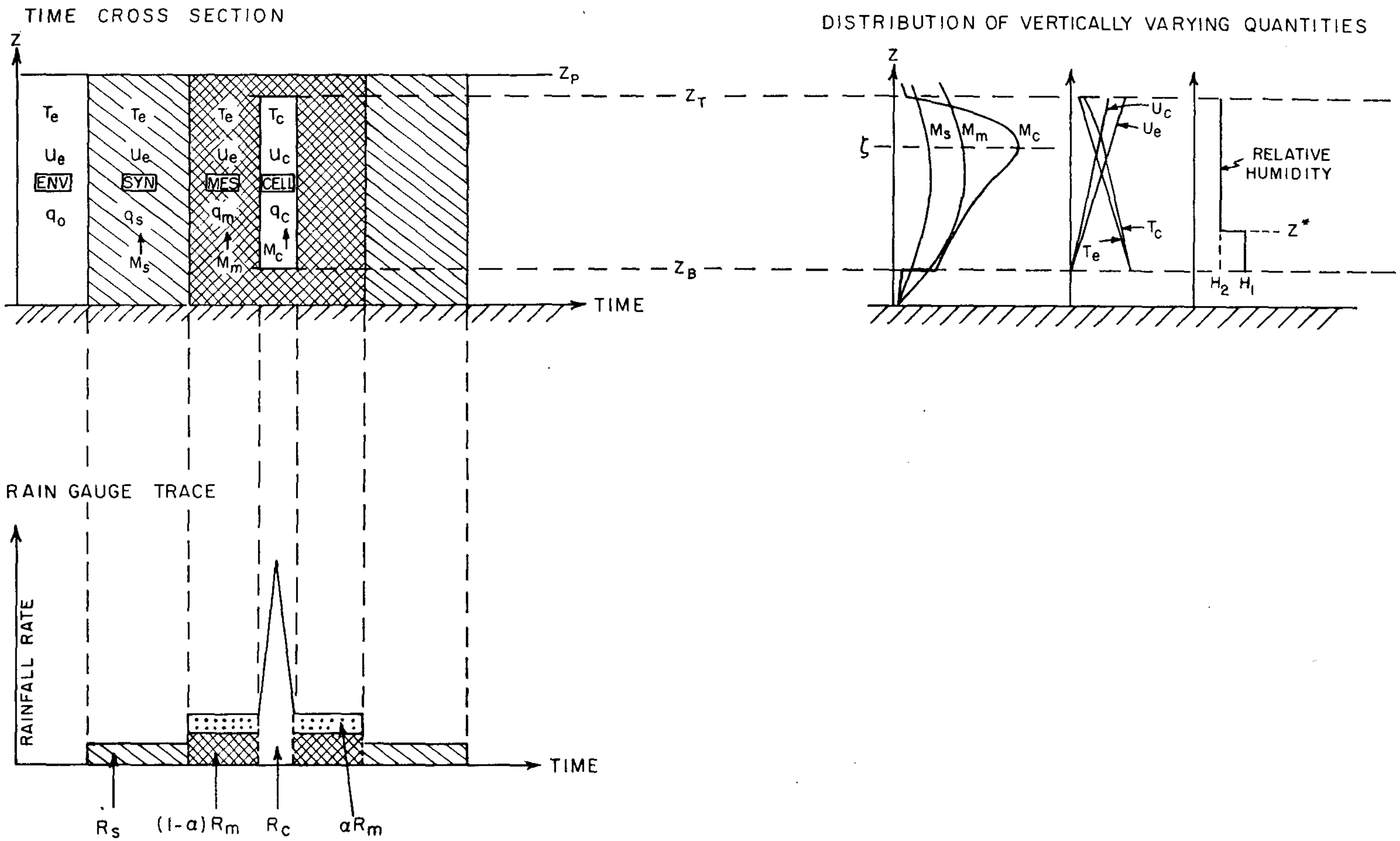

FIG. 1. Schematic diagram of model storm. The regions CELL, MES, SYN, ENV refer to cells, mesoscale areas, synoptic areas and environment, respectively; M's are masses of air transported upward within the different regions to produce the respective R's which are amounts of rain deposited per unit area at the station where measurements are made; α is the fraction of R_m assumed to *originate* in cells; and T is temperature, u westerly wind speed, and q water vapor mixing ratio. The relative humidity for the region ENV is represented ideally with H_1 being the mean value for low levels and H_2 the mean for upper levels. In the precipitating regions the air is saturated.

rected for the effects of downdrafts, are

$$\tau_{sh}=M_c(z)c_p[T_c(z)-T_e(z)], \qquad (2)$$

$$\tau_{am}=M_c(z)a\cos\phi[u_c(z)-u_e(z)], \qquad (3)$$

c_p being the specific heat of air at constant pressure, a the earth's mean radius, and ϕ the latitude.

It should be noted that (2) represents the direct transport of sensible heat due to the fact that throughout most of their depth the air in the updrafts is warmer than the air subsiding in the environment. The mechanism by which this heat reaches the environment is unspecified in our analysis which is concerned only with the magnitude of transports. The problem of the heating mechanism has been addressed by Braham (1952) and Gray (1971) who place emphasis on heating by compression in the region of compensating downward motion. While it is useful to think of the cumulus heating mechanism in such terms, it should be noted that the magnitude of the compressional heating outside cells, minus any evaporation of detrained condensate, must be equal to the convergence of (2) plus net latent heating (Yanai *et al.*, 1973).

Effects of lifting and entrainment may be considered in terms of the conservation of some quantity ξ in the portion of the cells bounded by the levels z and dz as shown in Fig. 2. The conservation is expressed by

$$M_c\xi_c-[M_c\xi_c+(dM_c)_\epsilon\xi_c-(dM_c)_\delta\xi_c+M_cd\xi_c] +\xi_e(dM_c)_\epsilon-\xi_c(dM_c)_\delta+M_cSdz=0, \qquad (4)$$

where products of differentials are neglected and S represents the change of ξ_c per unit depth per unit mass of air due to any source or sink between z and $z+dz$. In (4) the terms involving $(dM_c)_\delta$ disappear by cancellation reflecting the fact that exiting air cannot directly affect the dilution of ξ in the cell.

By rearranging terms, (4) becomes

$$\frac{d\xi_c}{dz}=E(\xi_e-\xi_c)+S. \qquad (5)$$

A relation between mass transport at any level, $M_c(z)$, and the condensate C is obtained by integrating the equation for continuity of water vapor through the depth of the cells. In (5), ξ is replaced by q and the term S is a sink equalling the amount of vapor condensed; as a result, we have

$$C=\int_{z_B}^{z_T} M_c\left[E(q_e-q_c)-\frac{dq_c}{dz}\right]dz. \qquad (6)$$

From (2), (3) and (6) we wish to compute M_c, τ_{sh} and τ_{am}. Some of the quantities which appear in the equations can be measured, but others must be specified by making assumptions based on empirical evidence or dynamic cell models.

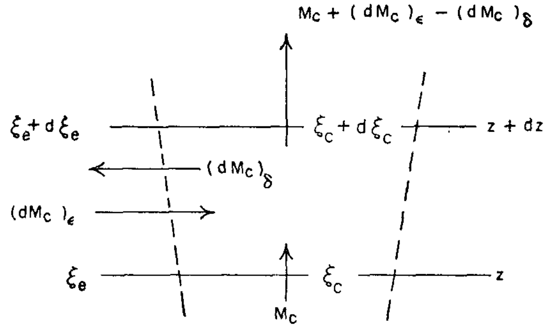

FIG. 2. Sketch illustrating entrainment, detrainment and vertical transport of quantity ξ in cumulus cells.

4. Measured quantities

Measured quantities are the amounts of rain falling in the cells, the depth of the layer containing the cells, and the distribution with height of wind, temperature and humidity in the larger scale environment.

The cells appear as small intense echoes on the plan position indicator of a radar instrumented for quantitative measurements or as brief sharp peaks on a rain gauge record which has high resolution in time. They are identifiable by their small dimensions, 2–4 km with a duration of 2 or 3 min over the rain gauge, and by the steep intensity gradients and relatively high rainfall rates within them. The value of R_c is obtained by summing over the various cells in the area or time of interest. Criteria for distinguishing between precipitation in the areas of different scales are based on the observations of Austin and Houze (1972) and are discussed more specifically by Houze (1973).

The top of a cell or cells, z_T, is obtained from the radar range-height indicator. The base of the cells, z_B, often is not evident on the radar as the falling precipitation causes echoes to extend below the base of the overturning layer. The cell base is assumed to be at a height of 1 km, roughly the top of the friction layer in temperate latitudes, unless soundings indicate a warm-frontal inversion or stable layer with the top at some other height.

Wind field, humidity and temperature in the general environment are obtained from conventional rawinsonde soundings.

5. Relation of cellular condensation to measured precipitation

The total amount of water condensed within cellular updrafts is expected to be considerably greater than R_c, the rain which falls directly from them. It is generally recognized that the efficiency of precipitation from cumulus convection is low (e.g., Houghton, 1968). Some of the cellular condensate is evaporated in downdrafts while some is left behind as the vertical air motions cease. Part of the condensate left behind may be evaporated at the edge of the storm or left aloft as

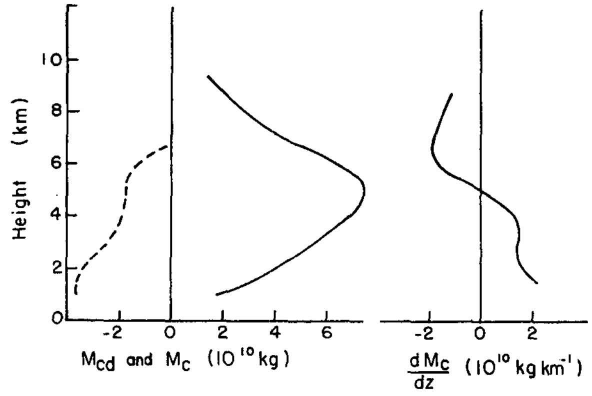

FIG. 3. Vertical mass transports in updraft and downdraft, and net entrainment for average air-mass thunderstorm in Ohio, plotted from results of Braham (1952).

cloud, while some becomes mixed into the surrounding mesoscale precipitation area where the hydrometeors may continue to grow and eventually fall as part of the mesoscale rain. Empirical evidence is sought for determining how the total cellular condensate is related to R_c in various types of storms.

Braham (1952) considered the water budget for an average cell in air mass thunderstorms in Ohio and Florida, and calculated that 19% of the condensate reached the ground as precipitation, 36% was left aloft as cloud, and 45% was evaporated in the downdraft. For these storms, the total cellular condensate is thus approximately $5R_c$.

Melvin (1968) analyzed the characteristics of thunderstorms in New England from quantitative radar data. He calculated the ratio of total precipitation to cellular precipitation for six thunderstorms, each of which contained from three to fourteen cells during its lifetime; values ranged from two to six with an average of four. In these storms, all of the precipitation was probably of cellular origin; it is also probable that some condensate was evaporated or left aloft as cloud. Thus, Melvin's results suggest that for this type of storm, on the average C is at least as large as $4R_c$.

Austin and Houze (1972) noted that cells are almost invariably located within small mesoscale precipitation areas, and suggested that the rain in these areas which exceeds the surrounding precipitation results from condensation in the cells. From their observations, which cover 24 small mesoscale areas in nine storms, the ratio of the excess precipitation in each small mesoscale area to the cellular rain within it has been calculated. The median value for the nine storms is 2.5, the mean is 4.0, and the range 1–10 with only one exceeding 6. The storms in the group included a variety of types, but most of them were cyclonic storms with widespread precipitation.

In the two radar studies any moisture which was condensed in an updraft and subsequently re-evaporated in a downdraft would not be observed. Thus, the only information regarding the magnitude of this portion of cellular condensation is from Braham (1952), and it pertains particularly to air-mass thunderstorms. In view of the extreme sparsity of empirical evidence, it was decided to exclude effects of the downdraft during the calculations. In Section 9 consideration is given to estimating a correction factor to compensate for this omission.

The observations outlined above suggest that for most storms the ratio of C (excluding condensate which is evaporated in the downdraft) to R_c tends to be between 2 and 10 with an average value of about 3–4. On the basis of these results, we consider $C=3R_c$ to be a "best estimate" with an uncertainty of a factor of 3. Since C must be at least as great as R_c, the lower limit is definite; but an upper limit in the vicinity of 10 is an estimate for cumuli which produce measureable precipitation, since the absolute upper limit is infinite.

6. Specification of entrainment rate and mass transport profile

It has long been recognized that cumulus convection in the atmosphere must be accompanied by some entrainment in order to satisfy continuity and to explain observed vertical developments, temperatures and liquid water contents in cumulus clouds. In very small tropical cumuli, 1–2 km in depth, Stommel (1947) measured temperatures which implied entrainment rates of $\sim$1.0 km^{-1}. More recently, Sloss (1967) obtained values ranging from 0.1–0.4 km^{-1} in continental cumulus congestus clouds. Byers and Braham (1949) deduced from direct measurements of horizontal inflow that cumulus clouds growing into thunderstorms entrained on the average at a rate of $\sim$0.2 km^{-1}. They also presented results of thermodynamic calculations which showed that realistic thunderstorms cannot develop with entrainment rates $\gtrsim$0.5 km^{-1}.

Laboratory experiments on steady-state jets and bubbles have led to one-dimensional models of cumulus updrafts with the entrainment rate given by

$$E=\frac{\epsilon}{r}, \tag{7}$$

where r is the radius of the updraft and ϵ is a constant. Such models, with $\epsilon=0.2$, have been used successfully in predicting the vertical development of cumulus clouds (Simpson and Wiggert, 1969, 1971; Weinstein and MacCready, 1969). In these experiments, the cloud radii ranged from 0.4 to 2.4 km with corresponding assumed entrainment rates of 0.5 to 0.08 km^{-1}.

The available empirical and dynamical evidence then suggests an inverse relationship between entrainment rate and updraft dimension with values of E ranging from about 1.0 km^{-1} for small trade cumuli to 0.1–0.2 km^{-1} for large thunderstorms.

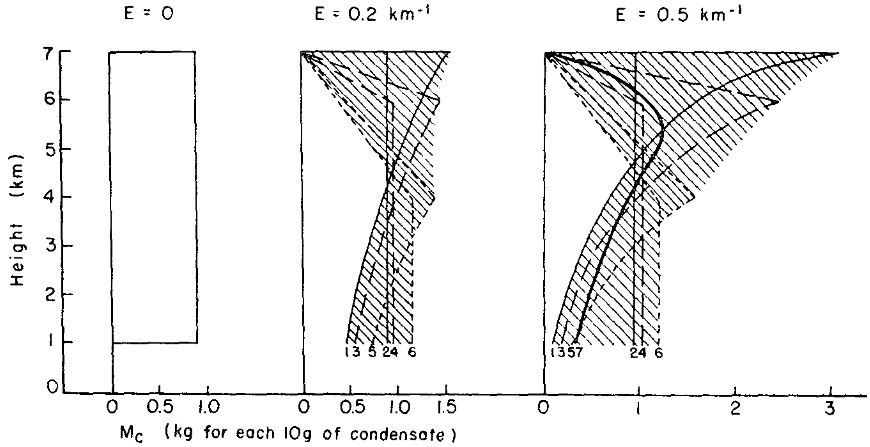

FIG. 4. Computed curves for vertical mass transport M_c, covering range of values for entrainment rate E, detrainment rate D, and height ζ. For all cases, condensate $C=10$ gm, q_c decreases linearly with height at the rate 2×10^{-3} km^{-1}, and $(q_c-q_e)=5\times10^{-4}$ at all levels. Labels on the curves have the following meanings for D below level ζ:

1. $D=0$, $\zeta=7$ km
2. $D=E$, $\zeta=7$ km
3. $D=0$, $\zeta=6$ km
4. $D=E$, $\zeta=6$ km
5. $D=0$, $\zeta=4$ km
6. $D=E$, $\zeta=4$ km
7. Profile used in model

In calculating the mass transport, we prescribe the profile of M_c and the entrainment rate; the magnitude of M_c is obtained from (6). For simplicity, the profile of mass transport is assumed to be continuous with minima at z_B and z_T and a single maximum at some height ζ. These general features appear reasonable in view of Braham's (1952) analysis of Thunderstorm Project data showing such a function with a maximum approximately halfway between cell base and top (Fig. 3). Diagnostic studies of regions of tropical convection imply that net outflow is located in the upper one-third to one-half of cells (Reed and Recker, 1971; Yanai *et al.*, 1973). Some one-dimensional models of cumulus clouds, such as the one of Squires and Turner (1962), suggest a maximum mass transport at the cell top with the outflow in an infinitesimally thin layer. On the basis of the above studies, the cell center and cell top are considered reasonable limits for the position of ζ.

The simultaneous specification of entrainment rate and the profile of M_c implies an assumed pattern of detrainment for a cell since, by definition,

$$\left(\frac{dM_c}{dz}\right)_\delta=\left(\frac{dM_c}{dz}\right)_\epsilon-\left(\frac{dM_c}{dz}\right). \qquad (8)$$

Little is actually known regarding the magnitude of detrainment. In laboratory jets and bubbles, there is no detrainment because the "in-cloud" fluid is more turbulent than the still environment so that fluid mixed at the interface is drawn into the bubble or plume. Similar behavior might be expected in cumulus clouds in the atmosphere provided there is very little wind. If the vertical shear is significant, however, the air rising in the updraft has a different horizontal speed from that in the environment; in this case, inflow from the upwind side and erosion of cloud air by the flow of the environment around it might cause larger values of both entrainment and detrainment than would occur in still air. The experiments in cumulus dynamics provide no insight into the amount of detrainment since it does not affect the dilution and thereby influence the height to which the cloud top ascends.

For physical consistency in our model calculations, the monotonic variation assumed for M_c above and below ζ must be such that detrainment is less than entrainment in the lower part of the profile while it exceeds entrainment in the upper region. These requirements limit considerably the range of profiles which can be assumed for M_c. In Fig. 4 are a number of mass transport curves computed from (6) for different values of entrainment rate, detrainment, and height of level ζ, but all corresponding to the same total condensate and the same distribution of temperature and humidity in the environment. These curves contain the range of reasonable values for E, ζ and detrainment which may be assumed for a cell 6 km in depth.

The curve which we use in model computations is shown in Fig. 4c and has the form

$$M_c(z)=M_c(\zeta)f_c(z), \qquad (9)$$

where

$$f_c(z)=\frac{2}{(2-E)}\exp[E(z-\zeta)]-\frac{E}{(2-E)}\exp[2(z-\zeta)], \quad \text{for } z_B<z<\zeta, \qquad (10)$$

with lengths being expressed in km and E in km^{-1}. Above ζ, M_c decreases parabolically to zero or to some

TABLE 1. Vertical velocities w_{max} considered realistic for model cells at height of maximum updraft.

Cell depth (km)	w_{max} (m sec^{-1})
1–3	3
3–5	4
5–7	5
7–9	6
9–11	7
11–13	8
13–15	9

small finite value corresponding to the larger scale lifting outside the cells. The level ζ is placed at $z_B+\frac{3}{4}(z_T-z_B)$. The entrainment rate is assumed to be

$$E=\frac{0.2}{0.13(z_T-z_B-1)}. \tag{11}$$

There is some physical basis for the forms of (10) and (11). The dominant term in (10) shows an exponential increase with height as would be expected for a cell of constant radius and constant entrainment rate, and (11) indicates an inverse relation between entrainment rate and cell dimension with the value of E covering the range 1.0 to 0.1 km^{-1} for cells 2.5 to 16 km in height. We emphasize, however, that the important feature of (10) is neither its particular mathematical form nor its physical implications but the facts that it lies comfortably within the shaded regions in Fig. 4 and that it has a similar shape for cells of any height. Only in shallow layers near the top and bottom and for extreme values of assumed entrainment or detrainment is there more than a factor of 2 between either limit of the shaded area and the model curve. Since the shaded areas represent the entire gamut of assumptions about entrainment, detrainment and height of maximum mass transport which are compatible with the empirical evidence and dynamic models, a mass transport curve of the selected shape *or any other shape which is roughly similar to it* should not differ from the actual mass transports by more than a factor of 2 at any level. The

TABLE 2. Peak updraft velocities measured in cumulus clouds.

Cloud depth (km)	Vertical velocity (m sec^{-1})	Investigators	Method of observation
0.7–4.0*	3.5–6.5*	Warner (1970)	Aircraft penetration
1.5–5.0*	0–7*	Cunningham *et al.* (1964)	Cloud photography and aircraft penetration
12**	5–7**	Byers and Braham (1949)	Aircraft penetration
15†	23†	Barnes (1970)	Balloon ascent

* Range.
** Average.
† Single observation.

selected curve is a little nearer to the lower limits than the upper ones; therefore, if the error should exceed the estimated uncertainty of plus or minus a factor of 2, the result would be an underestimate rather than an exaggeration of the transports.

7. Temperature and humidity differences between updrafts and immediate invironment

For computing the vertical transport of sensible heat from (2), the temperature difference between cell air and environment is required. Also, to integrate (6) it is necessary to know q_c, which depends on T_c, at each height.

The temperature excess (T_c-T_e) needed to produce cells of any observed depth can be computed from equations used in one-dimensional dynamic cell models provided the lapse rate of temperature and the vertical velocity are postulated. For this purpose the cellular lapse rate is assumed to be moist adiabatic, characterized by equivalent potential temperature[2] Θ_E, and the vertical velocity w_c is given the same profile as the mass transport function. Thus,

$$w_c=w_{max}f_c(z), \tag{12}$$

with $f_c(z)$ as in (10). Values of w_{max} for cells of different depths, as specified in Table 1, are based on the measurements summarized in Table 2.

The equation for the continuity of water in a layer of depth dz is

$$\frac{d\mu}{dz}=A\left[-\frac{dq_c}{dz}+E(q_e-\mu-q_c)\right], \tag{13}$$

where μ is the mixing ratio of water in the form of hydrometeors, the quantity in brackets is the total water condensed per unit height, and A is the fraction of this condensate retained by the rising mass of air, the remainder being assumed to fall out as precipitation.

For the vertical acceleration we use an equation which is similar to one developed by Levine (1965) and applied by Simpson and Wiggert (1969, 1971) in field experiments; it is

$$w_c\frac{dw_c}{dz}=-w_c^2E+\frac{g}{1+\gamma}\left[\frac{T_c^*(1+\mu)^{-1}-T_e^*}{T_c^*(1+\mu)^{-1}}\right], \tag{14}$$

where γ is the apparent mass coefficient due to acceleration of the fluid surrounding the rising cloud element, and T_c^* and T_e^* are the virtual temperatures in the cloud and its surroundings. Values of μ and (T_c-T_e) have been computed for a number of cell depths and a number of values of Θ_E by a simple stepwise integration of (13) and (14) with μ set to zero at cell base. Whenever

[2] It is recognized that in actual cells the lapse rate is not likely to be exactly moist adiabatic, but the concern here is not with the cell temperatures per se but with the temperature *excess* over the environment which is needed to produce cells of an observed depth.

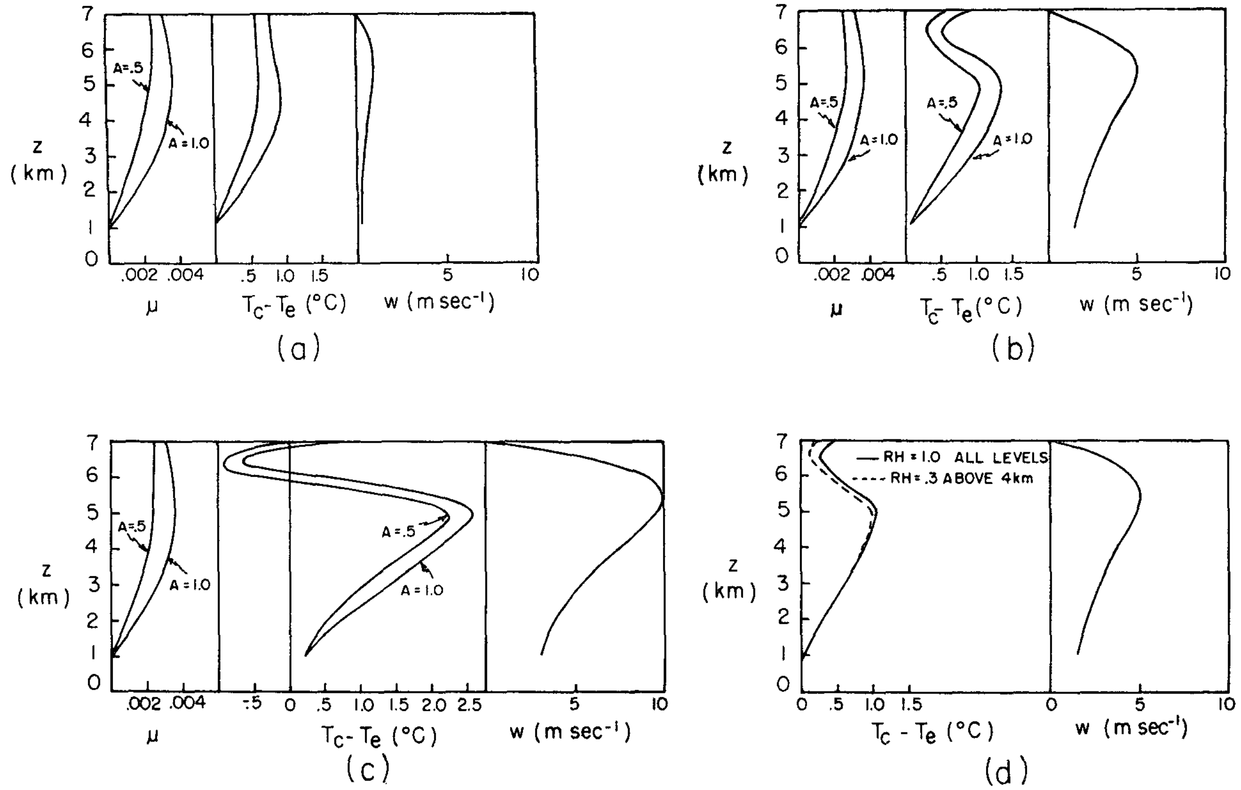

FIG. 5. Calculations of (T_c-T_e) and μ for different values of updraft velocity w_c, precipitation parameter A and relative humidity. In (a)–(c) both cell and environment are assumed to be saturated.

a value of (T_c-T_e) is called for in calculating the transports, reference is made to the results of these calculations with Θ_E in any given storm being approximated as the mean potential equivalent temperature shown by soundings for the layer containing the cells.

Relative magnitudes of uncertainties associated with assumptions about updraft speeds and values of the factor A are shown in Fig. 5; (T_c-T_e) differs by roughly a factor of 3 when w_{max} is changed by an order of magnitude, and the difference for $A=1$ and $A=0.5$ is only 30% at its maximum value. Similar computations have shown that a variation of γ between 0 and 0.5 has negligible effect on the computed temperature difference. Cells are modelled as having a saturated environment because they usually occur within mesoscale precipitation areas. On occasion, however, they may extend above the surrounding precipitation. Fig. 5c shows that reducing the relative humidity in the environment from 100% to 30% above the 4-km level in a cell which extends to 7 km has no significant effect on the computed temperature difference.

It can be seen from Fig. 5 that computed values of (T_c-T_e) are more sensitive to the vertical velocity than to the other assumed quantities. The measurements in Table 2 are useful for making empirical estimates of w_{max} for cells of different heights, but they are too few to permit computation of probable error. Their ranges suggest, though, that it is very unlikely that actual values would differ from the assumed ones by more than a factor of 3–4. Since an order of magnitude change in w_{max} caused a change of less than a factor of 3 in (T_c-T_e), except in a shallow layer near the cell top, we estimate the uncertainty in the computed values of (T_c-T_e) to be plus or minus a factor of 2.

8. Difference in the horizontal wind speed in cells and in environment

For the vertical transport of angular momentum, we consider only the east-west component of the horizontal wind. Within the cells, two factors contribute to the horizontal momentum: 1) the momentum of the air which is entrained and mixed tends to be conserved; 2) to the extent that a difference exists between u_c and u_e, the outside air exerts a pressure or drag force on the rising air. This drag force has been discussed by Malkus (1952), Hitschfeld (1960) and Bates and Newton (1965), and is estimated by

$$S=\frac{C_D}{\pi r w_c}(u_e-u_c)^2, \tag{15}$$

where C_D is a dimensionless drag coefficient which would be zero if environmental air were not deflected at all by

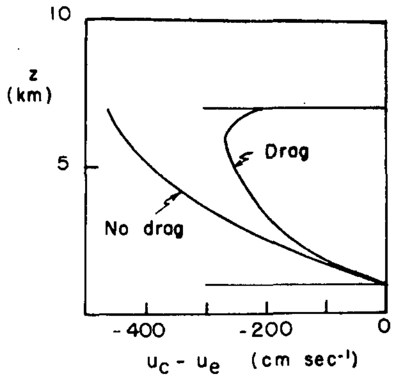

FIG. 6. Computed values of difference in horizontal wind speed inside and outside of cells (u_c-u_e) for two extreme values of drag coefficient.

the updraft and would be unity if the updraft acted as a solid cylinder embedded in the flow. The drag force acts as a source term in (5) which, when ξ is replaced by u, may be written

$$\frac{du_c}{dz}=E(u_e-u_c)+\frac{C_D}{\pi r w_c}(u_e-u_c)^2. \tag{16}$$

The radius of the updraft is calculated from the cell depth by the implicit relation in (7) and (11). With the vertical velocity specified as in Section 7, (16) is solved for (u_c-u_e) with $u_e=u_c$ at the cell base. The uncertainty associated with modelling the pressure drag is illustrated in Fig. 6 which shows the values for the same cell computed for $C_D=0$ and for $C_D=1$. The largest difference is near the top of the cell and is less than a factor of 2 at all levels. The appropriate value for C_D is not known; consistent with our policy of being conservative in computing the transports, we set it equal to 1.

9. Effects of cellular downdrafts

Effects on the transports due to downdrafts inside of cells have not been included in (2) and (3) because we have too little empirical evidence to relate them explicitly to the measured quantities. A rough estimate for the case of air-mass thunderstorms can be made from Braham's (1952) results which indicate that on the average 45% of the total condensate is re-evaporated in the downdraft. Inclusion of this portion of the condensate would increase the average value of C from $3R_c$ to $5R_c$, thereby increasing the estimated transports at all levels by 67%. Furthermore, if the temperature and momentum differences relative to the environment are of approximately the same magnitude but opposite in size in updrafts and downdrafts, the downdraft itself would add an equivalent amount to the transports in the lower half of the cell where it occurs.

It was further noted by Braham (1952) that less intense cells, ones with total rain $<10^7$ kg, often failed to exhibit evidence of downdrafts. For the weaker cells, therefore, appropriate correction factors may be considerably smaller than those noted above.

10. Estimates of uncertainties

Uncertainties in the measured quantities can be adequately evaluated only for particular sets of data. In general, a well-calibrated radar can measure instantaneous rainfall rates within about a factor of 2; by averaging over a number of storms or cells the accuracy be improved to perhaps ±50%. Rain gauge records indicate within a few percent the amount of rain falling at a point, but sample randomly in individual cells; therefore, a long time period (seasons or years) is required to obtain a representative sample of convection in a given area.

In the foregoing sections, assumptions made in calculating the quantities M_c, (T_c-T_e) and (u_c-u_e) are discussed, and the associated uncertainties are evaluated by considering limiting cases and computing ranges of values. The principal uncertainties are:

Source of uncertainty	Estimated magnitude
Estimation of cellular condensation C from precipitation measurements	A factor of 3 in M_c
Specification of the vertical variation of M_c and entrainment	A factor of 2 in M_c at any level
Specification of vertical velocity in temperature calculations	A factor of 2 in (T_c-T_e)
Modelling of horizontal drag on cells	Less than a factor of 2 in (u_c-u_e)

Combining these uncertainties as if each were independent yields roughly plus or minus a factor of 4 for the cumulus-scale transports of sensible heat and angular momentum computed by Eqs. (2) and (3). It should be recognized, however, that in the case of the momentum transport, there is definitely a tendency for errors to counteract each other. That is, assumptions that lead to an overestimate of M_c (i.e., too much entrainment or mixing) will simultaneously lead to an underestimate of (u_e-u_c), so that the product of the two is affected only slightly.

In view of the omission of the effects of downdrafts and our generally conservative choice of parameters such as the ratio C/R_c and the drag coefficient C_D, transports computed by the proposed method would tend to underestimate the true values. Any overestimate should not exceed a factor of 4, but underestimates might be as large as an order of magnitude.

11. Examples of the computations

Figs. 7 and 8 show examples of radar and rain gauge data and some computed transports. Computations were based on gauge data and represent transports for the amount of cellular rain that fell on one square centimeter. For these computations the environmental wind was approximated by a linear function in height with the indicated shear.

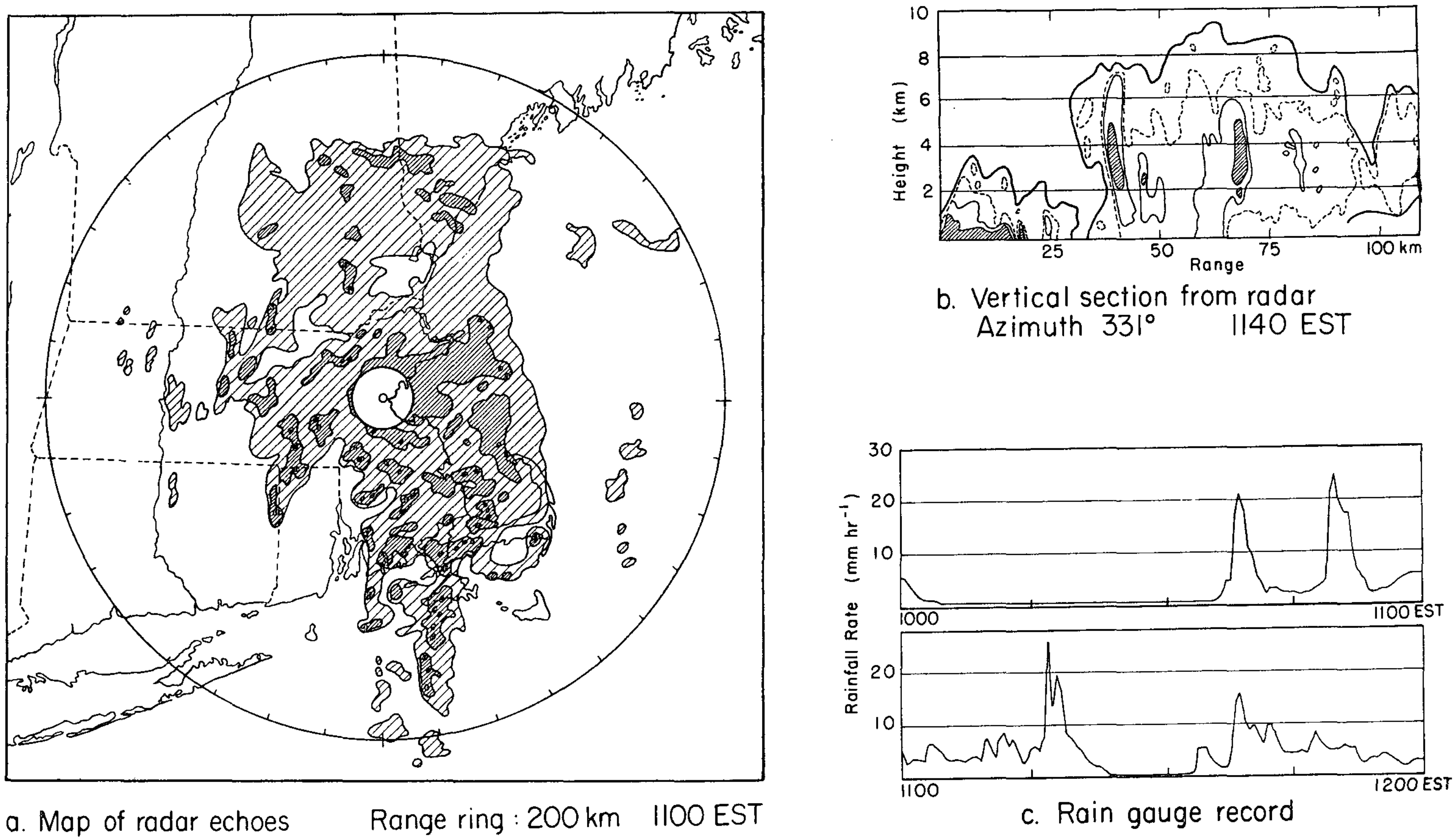

FIG. 7. Examples of high-resolution rainfall data from WR-66 radar ($\lambda = 10$ cm, beamwidth $= 1.3°$) and tipping-bucket rain gauge for 25 March 1969. Thresholds of intensity levels on the radar map and the three highest levels on the vertical section correspond to equivalent rainfall rates of 3, 8 and 25 mm hr^{-1}. Gauge is located at azimuth 290° and range 28 km relative to the radar.

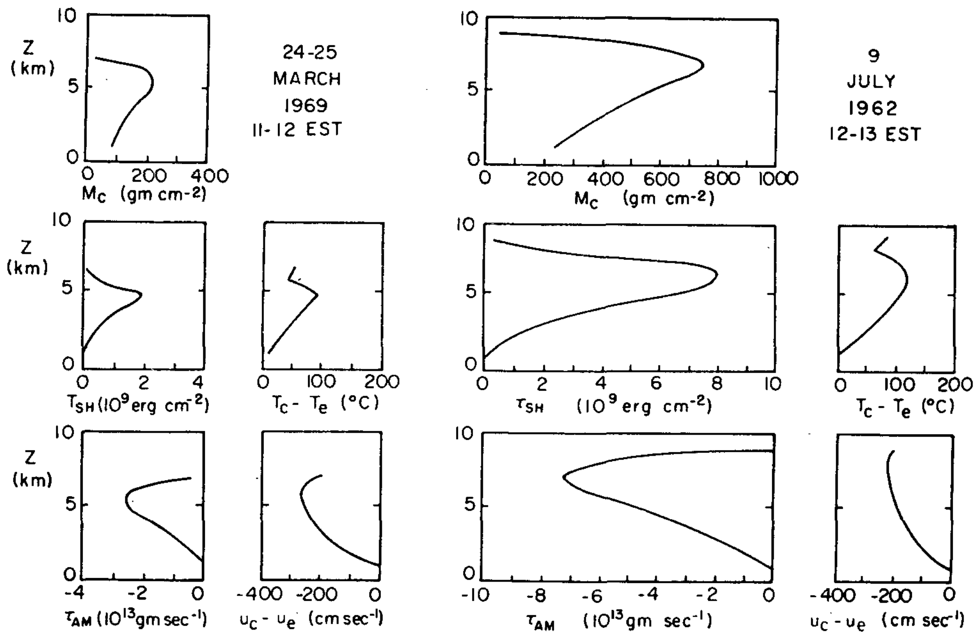

FIG. 8. Model cumulus-scale vertical mass transport M_c, sensible heat transport τ_{sh}, angular momentum transport τ_{am}, associated model temperature difference, $T_c - T_e$, and westerly wind speed difference, $u_c - u_e$, between cells and their environment for two storms at West Concord, Mass. Input quantities for the March storm are $z_B = 1$ km, $z_T = 7$ km, $\Theta_E = 310$K, $R_c = 0.33$ cm, shear $= 1.7$ m sec^{-1} km^{-1}. Input quantities for the July storm are $z_B = 1$ km, $z_T = 9$ km, $\Theta_E = 335$K, $R_c = 1.83$ cm, shear $= 0.9$ m sec^{-1} km^{-1}.

During the hour 1100–1200 EST 25 March 1969, a number of cells passed over the gauge as shown in Fig. 7. On 9 July 1962 between 1200 and 1300 EST a thunderstorm complex with several intense cells passed over the gauge.

12. Use of transport computations in meteorological research

At the present time, very few quantitative radar data suitable for this type of analysis are available; therefore, the applications which have been made rely on rain gauge records to provide rainfall amounts with the radar being used to indicate cell depths. Houze (1973) has made a climatological study of the vertical transports of angular momentum and sensible heat by cumulus-scale convection. His comparison with transports accomplished by larger scale circulations indicates that the cumulus-scale effects are significant in the global heat and momentum budgets in temperate latitudes as well as in the tropics. In an earlier study, Tracton (1968) used a simpler version of the same general technique, as originally suggested by Austin (1968), to study the vertical transports in a deep extratropical cyclone. In the selected cyclone, cumulus convection appeared to be the major mechanism for the vertical transport of momentum in the lower troposphere (700 mb), while the synoptic-scale transport dominated at 500 mb.

Methods of taking and processing radar measurements which will make them amenable to digital analysis are now becoming operational. As the data become available, this technique can be used to assess the role of cumulus-scale convection in specific events, thereby contributing to our understanding of the relative importance of circulations of various scales in the overall energy and momentum balance.

As understanding of the processes of cumulus convection increases and the limits of reasonable assumptions about efficiency, entrainment and downdrafts become narrower, uncertainties in calculations of transports accomplished by precipitating cumuli will be commensurately reduced. At the same time, computations of the type described in this paper may well serve as a means of gaining further insight into the processes of small-scale convection. When adequate measurements of various types are acquired during the same event (e.g., during GATE), comparison can be made of the cumulus-scale transports computed from rainfall amounts, from satellite photographs, and from diagnostic models which use sounding data and parameterization schemes to determine the cumulus-scale effects. Such comparisons should be very useful in assessing the validity of assumptions in the various approaches and thereby may cast light on the nature of the processes which are involved.

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