Errata for
Numerical Methods for Wave Equations in Geophysical Fluid Dynamics

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Preface

- *p. xi, last line:* replace "artisically" by "artistically"

Chapter 1

- *p. 23, line 7:* delete "then be"
- *p. 24, 3rd text line from bottom:* delete the phrase "indexanelastic approximation"
- *p. 25, 13th text line from bottom:* "$c_s \to 0$" should be "$c_s \to \infty$"
- *p. 25, left side of both of the last two displayed equations:* $(\omega - Uk^2)$ should be $(\omega - Uk)^2$
- *p. 31, line 9:* "(1.67) and (1.68) form" should be "(1.66) forms"

Chapter 2

- *p. 38, 6th line from bottom:* "$\psi($" should replace "$\psi(($"
- *p. 40, line 4:* Replace "if in the" with "if, for any fixed value of $n\Delta t$, in the"
- *p. 41, line 6:* "criteria" should be "criterion"
- *p. 41, text line 17:* replace "the solution" by "the numerical solution"
- *p. 42, third line after (2.22):* replace "$\mu \leq 0$" with "$\mu < 0$"
- *p. 42, fourth line after (2.22):* replace "$\mu \geq 1$" with "$\mu > 1$"
- *p. 45, lines 22 and 23:* "Fredrichs" should be "Friedrichs"

- *p. 44, line 4:* Replace “linear” with “linear two-time-level”
- *p. 44, line 6:* Replace “such that” with “such that, for all sufficiently small Δt and Δx,”
- *p. 51:* Delete the first equation after (2.35) and the phrase “to yield” that follows this equation.
- *p. 57:* Replace $\kappa^2 \Delta t^2 - 1)|^{1/2}$ with $\kappa^2 \Delta t^2 - 1)^{1/2}|$
- *p. 58:* Delete text line 20, which begins “to yield $\phi^n = \ldots$”. Remove the paragraph break at the start of the following line (text line 21).
- *p. 66, middle line of displayed equation:* “$F(\phi_1), -$” should be “$F(\phi_1) -$”
- *p. 71, displayed equation:* “$e^{i\kappa k \Delta t}$” should be ‘$e^{i\kappa \Delta t}$”
- *p. 80, eqn. (2.71):* Δx^2 should be $(\Delta x)^2$
- *p. 81, first eqn.:* Δx^2 should be $(\Delta x)^2$
- *p. 81, second eqn.:* Δx^3 should be $(\Delta x)^3$, and Δx^4 should be $(\Delta x)^4$
- *p. 81, third eqn.:* Δx^4 should be $(\Delta x)^4$
- *p. 92, middle:* replace

$$c^* = \frac{\Delta x}{\Delta t} = c \qquad \text{by} \qquad \frac{\omega}{k} = \frac{\Delta x}{\Delta t} = c$$

- *p. 94, 6th line from bottom:* replace “$O\left[(\Delta x)^2\right]$ and numerical dispersion of $O\left[(\Delta x)^3\right]$” by “$O(\Delta x)$ and numerical dispersion of $O\left[(\Delta x)^2\right]$
- *p. 95, line 2:* replace “(2.99)” by “(2.95)”
- *p. 95, (2.100):* change the denominator in the last term to ∂x^2 so that this term contains the factor

$$\frac{\partial^2 \psi}{\partial x^2}$$

- *p. 98, 3rd displayed equation:* replace

$$u\frac{\partial^2 \psi}{\partial x^2} + v\frac{\partial^2 \psi}{\partial y^2} \quad \text{with} \quad u^2\frac{\partial^2 \psi}{\partial x^2} + v^2\frac{\partial^2 \psi}{\partial y^2}.$$

- *p. 98, 4th text line:* replace “by a second-order spatial difference” with “by spatial differencing of at least first-order accuracy”
- *p. 98, 8th text line:* replace “by second-order spatial differences” with “by finite differences”
- *p. 98, last displayed equation:* replace $\tilde{\mathbf{v}}_j$ by $\mathbf{v}_j^n$.

- *p. 99, Eqn. (2.109):* replace $(1-\mu)(\Big($ with $(1-\mu)\Big($
- *p. 99, first line after (2.109):* Replace "$O\left[(\Delta t)^2\right] + O\left[(\Delta x)^2\right]$ accurate" with "accurate to $O\left[(\Delta t)^2\right] + O(\Delta t \Delta x) + O\left[(\Delta x)^2\right]$"
- *p. 102, line 1:* "Fredrichs" should be "Friedrichs"
- *p. 102, Problem 6:* delete the equation number "(1)"
- *p. 102, Problem 6, 2nd eqn.:* Δx^2 should be $(\Delta x)^2$
- *p. 103, Problem 10. line 3:* replace "given" with "derived"
- *p. 105, last sentence of Problem 20:* replace this last sentence by " As before let $u(0) = 1$, $v(0) = 0$, and integrate each solution to time $t = 4$. Try choosing Δt to span the interval $[0, 4]$ in 13, 16, 20, 30, 50 and 100 steps. Discuss the relative performance of the two methods as a function of $f\Delta t$. Submit plots showing u and v as a function of t as computed by each method for the cases that use 13, 20 and 50 steps."
- *p. 105, Problem 21:* Note: the specific parameters in this problem are tuned to provide interesting results using single precision FORTRAN code. If you use matlab you will be computing with double precision on most computers (about 16 digits), and random numbers in the range $[-5 \times 10^{-7}, 5 \times 10^{-7}]$ should be added to the inital data to insure the timely development of the most unstable modes in the numerical solutions.
- *p. 106, last line of Problem 22:* replace "(7.23)" by "(2.99)"
- *p. 106, 3rd to last line of Problem 23:* replace "all three" by "both"

Chapter 3

- *p. 109, 2nd line from bottom:* replace "$(3+\sqrt{5})/2$" with "1.618", which is its numerically evaluated square root.
- *p. 113, line 11 from bottom:* replace "it not necessary" with "it is not necessary"
- *p. 126, first eqn.:* first two terms in parentheses are

$$u^2\frac{\partial\psi}{\partial x^2} + v^2\frac{\partial\psi}{\partial y^2}$$

- *p. 128, 2nd text line from bottom:* replace "of the right" with "of Δt times the right"
- *p. 129, (3.52):* add "$-$" to the right sides of both eqns.

- *p. 130, 2nd equation:* add a factor of $\psi(0)$ to the two middle expressions to obtain

$$\psi(t+\Delta t) = \exp[(\Delta t + t)\mathcal{L}]\psi(0) = \exp(\Delta t\mathcal{L})\exp(t\mathcal{L})\psi(0) = \exp(\Delta t\mathcal{L})\psi(t).$$

- *p. 134, first line after (3.65):* replace "Wave solutions" with "Oscillatory solutions"
- *p. 134, right side of last equation:* replace icv^s by icu^s
- *p. 146, 4th displayed equation line:* replace $|A_0| =$ with $|A_0|^2 =$
- *p. 147, 2nd to last line:* replace "(A.2)" by "(A.1)"
- *p. 148, line 8:* replace "such as advection" by "such as in the problem of advection"
- *p. 148, line 2 from bottom:* replace "will represented" by "will be represented"
- *p. 150, 4th line from bottom:* replace "wind speed" by "velocity"
- *p. 155, (3.104):* replace with

$$\frac{d\mathbf{u}}{dt} = \mathbf{A}\mathbf{u}$$

- *p. 156, 5th line:* replace "(3.106) and (3.107)" with "(3.105) and (3.106)"
- *p. 156, 6th text line from bottom:* replace "(3.107)" with "(3.105)"
- *p. 158, last eqn:* insert "$-$" after the first equal sign
- *p. 167, Problem 2, first line:* replace "unstaggered" with "staggered"
- *p. 168, Problem 5:* Replace "Show that" with "Show that, at least for some combinations of U and V,"
- *p. 168, Problem 7:* the last term inside the parentheses in the equation should be

$$M^2\frac{\partial^4\phi^n}{\partial x^4}$$

- *p. 170, Problem 13, first line:* replace "Compute exact and" with "Compute"
- *p. 171, end of subproblem (b):* add "Compare the numerical solutions with the exact solution determined in (a) at time $t = 3$."
- *p. 178, first eqn:* add a factor of i to the denominator so the result reads

$$\frac{e^{i(n-m)x}}{i(n-m)}$$

Chapter 4

- *p. 174, line 22:* Replace "(3.1)" with "(4.1)"
- *p. 176, 8th line from bottom:* Replace "are obtained" with "may be easily obtained"
- *p. 184, line 21:* $\kappa = ck$ should be $\kappa = -ck$
- *p. 192, last text line:* Replace e^{ikx_j} by e^{-ikx_j}
- *p. 195, line 10:* Replace "onethird" with "one third"
- *p. 199, caption to Fig. 4.5:* Replace "rather than a rhombus" with "with unequal adjacent sides, i.e., a rhomboid"
- *p. 201, text lines 9-10:* Replace "frequencies of the highest meridional wave numbers are" with "frequency associated with the highest total wave number is"
- *p. 210, first eqn.:* Inside the integral $\left(\frac{\partial\chi}{\partial\lambda}\right.$ should be $\frac{1}{a^2}\left(\frac{\partial\chi}{\partial\lambda}\right.$
- *p. 212, eqn. (4.86), 2nd line:* Change sign on last term to read

$$\ldots \chi_{m,n-1} + (n+1)(n+2)\epsilon_{m,n+1}\chi_{m,n+1}$$

- *p. 212, text line 12:* Replace "(Machenhauer 1979)" with "Machenhauer (1979)"
- *p. 213, text line 14:* Replace "that the error be orthogonal to the residual, or" with "that the residual be orthogonal to each expansion function, or"
- *p. 217, first eqn.:* Replace $\frac{1}{6}\mu\nu(k\Delta x)^3$ with $\frac{1}{6}i\mu\nu(k\Delta x)^3$
- *p. 222, (4.100):* change to

$$\frac{c^*}{c} = \frac{\sin\theta}{\theta\left(1+\sin^2\theta\right)}\left(-2\cos\theta \pm \left(9+\sin^2\theta\right)^{1/2}\right)$$

- *p. 223, Figure 4.8:* the plot of the computational mode is incorrect. The figure should appear as shown in Fig. 1.
- *p. 223, text line 4 through end of the first paragraph:* Replace text "The adjectives 'physical' . . . at poor spatial resolution." with:

  As discussed by Cullen (1982), the set of computational modes and the set of physical modes are, in fact, identical—so neither the physical nor the computational mode is spurious. The wave propagation characteristics of the quadratic-finite-element solution can be more clearly understood by multiplying (4.99) by $2/(1+r_{\mathrm{a}})$ and expressing the result as

$$q_n(t) = e^{ik\left[n\Delta x - c_{\mathrm{p}}^*(k)t\right]} + \beta e^{i(k-\pi/\Delta x)\left[n\Delta x - c_{\mathrm{n}}^*(k-\pi/\Delta x)t\right]}.$$

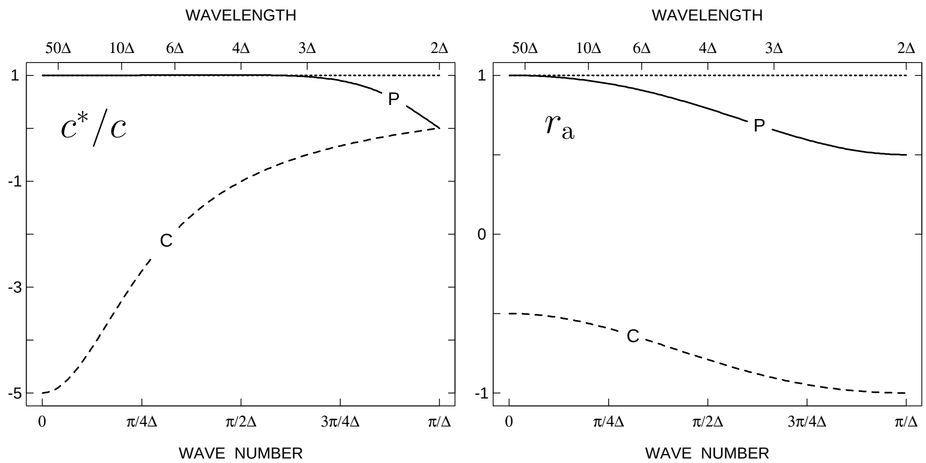

Figure 1: FIGURE 4.8: corrected.

Here q_n is the solution at the nth node, $c^*_\mathrm{p}(k)$ and $c^*_\mathrm{n}(k - \pi/\Delta x)$ are given, respectively, by the positive and negative roots in (4.100), $\beta = (1 - r_\mathrm{a})/(1 + r_\mathrm{a})$, and the value of r_a used to evaluate β is obtained by substituting $c^*_\mathrm{p}(k)/c$ into (4.101) (see www.atmos.washington.edu/˜durrand/QFEM.html).

- *p. 234, Problem 1, line 1:* Replace “$n > 0$” with “$n > 1$”.
- *p. 235, Problem 1, lines 1–2:* Replace “the significance of. . . wave.” with “how the change in sign of the wavenumber influences the relative phases of the correct and aliased waves at any mesh point where the wave amplitude is zero.”
- *p. 235, Problem 2, 4th text line:* Replace “If K is the number of modes retained . . .” with “If $2K + 1$ modes are retained . . .”.
- *p. 236, Problem 5b:* Replace with “Write down a trapezoidal approximation to the system of ordinary differential equations derived in (a). How would the efficiency with which this trapezoidal approximation can be computed change if ν did not depend on x?
- *p. 237, Problem 11, 3rd text line:* Replace sentence beginning“Use fast Fourier transforms . . .” with “Compute Fourier series expansions truncated at progressively higher wavenumbers K, such that $K = 2^m\pi$, for $m = 3, 4, \ldots, 8$.”

Chapter 5

- *p. 250, first line after (5.16):* Replace $f[\psi(j \pm \frac{1}{2})\Delta x)]$ with $f[\psi((j \pm \frac{1}{2})\Delta x)]$

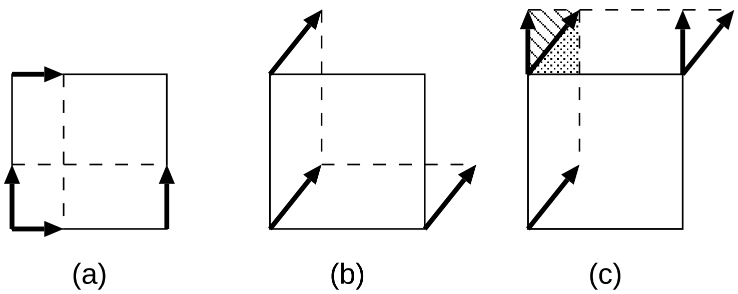

Figure 2: FIGURE 5.17: with corrected fill

- *p. 250, last equation:* Insert a factor of Δx into each of the first two terms so that the corrected equation reads

$$\sum_{j=j_1}^{j_2} \phi_j^{n+1} \Delta x = \sum_{j=j_1}^{j_2} \phi_j^n \, \Delta x + \Delta t F_{j_1-\frac{1}{2}} - \Delta t F_{j_2+\frac{1}{2}},$$

- *p. 253, caption:* "Fredrichs" should be "Friedrichs"
- *p. 254, lines 11, 15, 19 and 21:* "Fredrichs" should be "Friedrichs"
- *p. 275, line 16:* replace "not increase" with "not increase (and typically decreases)"
- *p. 275, line 17:* replace "kept TVD" with "kept sufficiently close to TVD"
- *p. 279, Fig. 5.17:* the stippled and diagonal fill did not reproduce properly. The figure should appear as shown in Fig. 2.
- *p. 283, line 13:* $r^n_{i-\frac{1}{2},j}($ should be $r^n_{i-\frac{1}{2},j} = ($
- *p. 283, line 15:* Add a multiplicative factor of R to the final term in the expression for $F^n_{i-\frac{1}{2},j}$
- *p. 288, caption:* Grid spacings are 0.02 and 0.01, not 0.2 and 0.1
- *p. 293, last equation:* both $A_{j+\frac{1}{2}}$ should be $F_{j+\frac{1}{2}}$
- *p. 298, line 15:* Add a multiplicative factor of R to the final term in the expression for $F^n_{i-\frac{1}{2},j}$
- *p. 299, last text line:* "Fredrichs" should be "Friedrichs"
- *p. 300, lines 4, 14:* "Fredrichs" should be "Friedrichs"
- *p. 300, Prob 10:* The first displayed equation should be

$$\sigma_j = \begin{cases} 0 & \text{if } ab \le 0, \\ \mathrm{sgn}(a)\ \min\big(2|a|, 2|b|, |a+b|/2\big) & \text{otherwise,} \end{cases}$$

Chapter 6

- *p. 305, 3rd text line from bottom:* "$x_{j-p} \leq x \leq x_{j-p-1}$" should be "$x_{j-p-1} \leq x \leq x_{j-p}$"
- *p. 308, line 21:* "$U\Delta t$" should be "$U\Delta t/\Delta x$"
- *p. 309, line 18:* Expressions should read $\alpha = U\Delta t/\Delta x - p$ and $\beta = V\Delta t/\Delta x - q$.
- *p. 309,* Subsection entitled *"Two Spatial Dimensions":* Change *all* superscripts "n" to "s" to distinguish them from the second spatial index.
- *p. 309, third text line from bottom:* replace ϕ_{N} with ϕ^s_{N}
- *p. 309, second text line from bottom:* replace ϕ_{NE} with ϕ^s_{NE}
- *p. 309, line 18:* replace $U\Delta t$ by $U\Delta t/\Delta x$ and replace $V\Delta t$ by $V\Delta t/\Delta y$.
- *p. 309, last equation:* the first line up to the left square bracket should read

$$\phi^{s+1}_{m,n} = \frac{\alpha}{2}(1+\alpha)\Bigg[$$

- *p. 314, text line 10:* replace "a partial" with "an ordinary" to read "(3.75) is an ordinary differential equation"
- *p. 314, text line 11:* replace "an ordinary" with "a partial" to read "(6.27) is a partial differential equation"
- *p. 314, text line 12:* replace "$2\tilde{x}^n_j - x_j$" with "$\check{x}^{n-1}_j$"
- *p. 315, Fig. 6.2:* replace with Fig. 3.
- *p. 320, third line after (6.36):* Should read $b^+ = h^0$.
- *p. 321, line 2:* replace $\tilde{\lambda} = \lambda e^{-iks}$ by $\tilde{\lambda} = \lambda e^{iks}$
- *p. 321, eqn. 6.39:* replace e^{iks} by e^{-iks}
- *p. 322, eqn 6.41:* replace $\frac{1}{2}\eta_0$ in the last term by η_0
- *p. 323, 3rd text line:* Should read $\tilde{H} = Hk\Delta t$.

Chapter 7

- *p. 343,* replace ωnt by $\omega n\Delta t$ in (7.19)
- *p. 351, text line 4:* replace ms^{-1} with m^{-1}.
- *p. 352, line 6:* Replace "(7.39)–(7.40)" with "(7.38)–(7.41)".

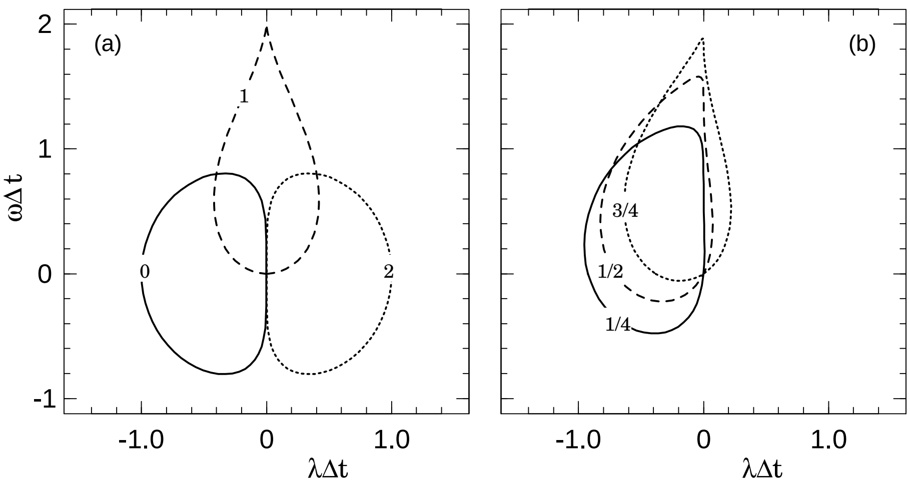

Figure 3: FIGURE 6.2: corrected.

- *p. 356, third eqn.:* Replace $\cos(\omega\Delta t)$ with $\cos^2(\omega\Delta t)$.
- *p. 357, 5th equation should be:*

$$h = P^{n-1} - \Delta t \left[c_s^2 \nabla \cdot \mathbf{v}^{n-1} + 2\mathbf{v}^n \cdot \nabla P^n \right].$$

- *p. 359:* Replace (7.63) with

$$\frac{\partial \psi}{\partial t} = \mathcal{L}(\psi).$$

- *p. 360:* Change "+" to "−" in (7.68)–(7.70)
- **p. 368, line 18 from bottom:* should read "$\ldots \Delta\tau = 0.625 \ldots$"
- *p. 373, last equation:* Replace $e^{i(kx+\ell z-\omega n\Delta t)}$ by $e^{i(kx+mz-\omega t)}$.
- **p. 375, eqn.7.97:* Replace $\nabla_z(\rho\mathbf{u})$ with $\nabla_z \cdot (\rho\mathbf{u})$
- **p. 375, eqn. 7.98* should read

$$\frac{dT}{dt} - \frac{\omega}{c_p\rho} = 0$$

- **p. 377, first equation after (7.103):* last term should be

$$\dot{\sigma}\frac{\partial(\,)}{\partial\sigma}$$

- *p. 384, first eqn.:* Lower limit of first integral should be z_s.
- *p. 385, (7.128):* Replace ∇_σ with $\frac{1}{g}\nabla_\sigma$.

- *p. 386, third eqn:* Replace $\mathbf{u}\dot{\sigma}\frac{\partial \mathbf{u}}{\partial \sigma}$ with $\mathbf{u} \cdot \dot{\sigma}\frac{\partial \mathbf{u}}{\partial \sigma}$.
- *p. 390, 2nd equation in (7.141) (for $\partial\delta/\partial t$):* Replace term βu with $\beta(U + u)$.
- *p. 390, 4th text line from bottom:* Replace β/c with $|\beta/c|$
- *p. 392, Problem 7:* Replace $\left\langle \langle \mathbf{u}_k \rangle^\sigma \dot{\sigma}_k \, \delta_\sigma \mathbf{u}_k \right\rangle^\sigma$ with $\left\langle \langle \mathbf{u}_k \rangle^\sigma \cdot \dot{\sigma}_k \, \delta_\sigma \mathbf{u}_k \right\rangle^\sigma$

Chapter 8

- *p. 401, 5th line of Section 8.1.3:* Replace "$0 \le x \le L$, and" with "$0 \le x \le L$, that $c > |U|$, and"
- *p. 402, 4th text line:* replace the sentence beginning with "This last form of the boundary . . ." with

  "Using the equation for the outward directed characteristic e [the second equality in (8.3)], this reduces to

  $$\left(\frac{\partial}{\partial t} + (U + c)\frac{\partial}{\partial x}\right)(u - u_e) = 0$$

  (Carpenter 1982)."
- *p. 402, 8th text line:* replace "eliminated from (8.5)" with "eliminated from (8.6)"
- *p. 402, eqn (8.12):* replace equation (8.12) and the following sentence with the equation and sentence:

  $$\left(\frac{\partial}{\partial t} + (U + c)\frac{\partial}{\partial x}\right)(\eta - \eta_e) = 0 \qquad (8.12)$$

  It is only the perturbation of the interior solution about the externally imposed values u_e and η_e that must be transmitted outward through the boundary.
- *p. 404, line 2:* Replace "nonphysical computational" with "nonphysical".
- *p. 408, lines 8, 10, 13 and 16:* Replace "computational" with "nonphysical".
- *p. 408, line 12:* Replace "The strong reflection at the right" by "The strong reflection at the left"
- *p. 410, 21st text line:* "Fig. 6.3a" should be "Fig. 8.3a"
- *p. 411, figure caption:* Replace "530" by "500", and "531" by "501"
- *p. 411, line 3:* Replace "$(0, 0)$" with "$(\theta, -\theta)$". Replace "both" with "all".

- *p. 414, line 6:* Replace "$\ldots x = L$. Then the ..." through the end of the paragraph with:

  $\ldots x = L$, and suppose that a wave of of the form $(u, v, h) = (\hat{u}, \hat{v}, \hat{h}) \exp[i(kx + \ell y - \omega t)]$ is approaching this boundary as shown in Fig. 8.4. The amplitudes $\hat{u}$ and $\hat{h}$ are related by the linearized x-momentum equation, which for the simplest case with no mean flow yields $\hat{u} = (gk/\omega)\hat{h}$. Thus after substituting for ω from the shallow-water dispersion relation for waves moving to the "east," the value of d at the right boundary may be expressed as

  $$d(L, y, t) = \left(\frac{gk}{\omega} - \frac{g}{c}\right) \hat{h} e^{i(kL + \ell y - \omega t)} = \frac{g}{c}\left(\frac{k}{(k^2 + \ell^2)^{1/2}} - 1\right) \hat{h} e^{i(kL + \ell y - \omega t)},$$

  implying that The condition $d(L, y, t) = 0$ cannot be satisifed without the simultaneous presence of a second reflected wave unless $\ell = 0$, or equivalently, unless the outward propagating wave is traveling at right angles to the boundary.

- *p. 417, third displayed equation:* Replace both the "w" by "ω" to obtain

  $$|r| = \left|\frac{\omega - kc}{\omega + kc}\right| = \left|\frac{1 - \cos\theta}{1 + \cos\theta}\right|,$$

- *p. 421, eqn. 8.43:* The exponent in the denominator of the last term should be 3/2 instead of 1/2.

- *p. 423, text line 7:* replace "defining $c = \omega/k$" with "recalling the definition $\tilde{c} = \omega/k$"

- *p. 423, third displayed equation:* both c should be $\tilde{c}$ and both c_j should be $\tilde{c}_j$

- *p. 423, text line 8:* c should be $\tilde{c}$

- *p. 423, text line 11:* c_j should be $\tilde{c}_j$

- *p. 434, first equation:* Replace $1 + \cos[8\pi \cdots$ with $1 - \cos[8\pi \cdots$.

- *p. 435, line 16 from bottom:* Replace $R_n \Delta t$ with $R_N \Delta t$.

- *p. 437, equation, Problem 2:* The $2\Delta x$ in the demominator should be be Δx.

- *p. 437, first equation of Problem 3:* The t and x should appear as subscripts as follows

  $$(\delta_{2t} + U\delta_{2x})\,\phi_j^n = 0$$

- *p. 438, Problem 8, line 3:* $\hat{P} = N\hat{w}/|m|$ should be $\hat{P} = N\hat{u}/|m|$

- *p. 438, Problem 8, line 5:* Replace "derive a partial" with "derive an approximate partial"

- *p. 438, line 4 from bottom:* "Problem 9" should be "Problem 8"

Appendix

- *p. 439, second line of equations from bottom:* change "$-$" to "$+$" in the last term of the numerator in the right equation to obtain

$$\delta_x^2 \phi_j^n = \frac{\phi_{j+1}^n - 2\phi_j^n + \phi_{j-1}^n}{(\Delta x)^2}.$$

Bibliography

- *p. 444:* Add reference: Carpenter, K.M., 1982: Note on the paper "Radiation conditions for the lateral boundaries of limited-area numerical models." *Quart. J. Roy. Meteor. Soc.*, **108,** 717-719.

- *p. 454, Temperton and Staniforth, 1987:* "two-time level" should be "two-time-level"

Index

- *p. 458, CFL entry:* "Fredrichs" should be "Friedrichs"

- *p. 459:* "Courant-Fredrichs-Lewy" entry should be "Courant-Friedrichs-Lewy"

- *p. 461:* "Lax-Fredrichs" entry should be "Lax-Friedrichs"