# Simulated equivalent reflectivity factor as currently formulated in RIP: Description and possible improvements

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## 1. Introduction

Read/Interpolate/Plot (RIP) is an extensive software package that reads mesoscale model output and allows for the creation and display of a wide array of diagnostic quantities. It was originally designed for use with the MM5 model, but is now compatible with both MM5 and WRF model output, and has been sufficiently generalized to be potentially compatible with the output from any mesoscale model.

One of the diagnostic quantities available in RIP is the field "dbz", which produces a simulated equivalent reflectivity factor based on the available mixing ratios of grid-resolved precipitation species. It is currently written to include rain, snow, and graupel. Because the algorithm is becoming increasingly utilized, not only by RIP users, but by others who have ported the algorithm to other applications, I am providing this description of the current algorithm, with some possible suggestions for improvement.

## 2. Background on equivalent reflectivity factor

Within the Rayleigh scattering range of drop sizes, the scattering of electromagnetic radiation by a drop is proportional to the sixth power of the drop diameter. Therefore, the scattering by a population of drops, per unit volume, is proportional to the *reflectivity factor*, $Z$, defined as the sixth moment of the drop size distribution:

$$Z = \int_0^\infty N(D) D^6 dD , \qquad (1)$$

where $N(D)$ is the number of drops per unit size range per unit volume, as a function of diameter $D$ of the assumed spherical drops. If a radar pulse illuminates a particular sample volume of the atmosphere, the radar equation [not shown here—see, for example, Eq. 4.3 in Houze (1993)] allows one to derive $Z$ within that sample volume as a function of the power returned to a radar, the distance of the sample volume, physical aspects of the Rayleigh scattering of water droplets, and various characteristics of the radar.

A sample volume can contain a quantity of ice particles which have different reflective properties than liquid water droplets. However, since the fraction of ice particles is generally unknown, the radar equation assuming liquid water properties is still used to derive an "equivalent reflectivity", $Z_e$, from the power returned. $Z_e$ is equal to $Z$ only for liquid water spheres (rain drops)—it is not the true sixth moment of the size distribution if ice particles are present. However, because of the simplicity of its calculation from returned power, $Z_e$ is the quantity generally derived and shown in radar echo displays.

In modeling applications, if one wants to compare the radar-observed precipitation field (expressed as $Z_e$) to the model's simulated precipitation field, an algorithm is required for converting the simulated precipitation mixing ratio fields and assumed particle shape, density, and size distributions to $Z_e$, so that an "apples to apples" comparison can be made. This is the purpose of the "dbz" algorithm in RIP.

## 3. Assumptions about particle shapes, densities, and size distributions

In calculating $Z_e$, the RIP algorithm makes assumptions consistent with those made in an early version (ca. 1996) of the bulk mixed-phase microphysical scheme in the MM5 model. For each species:

1. Particles are assumed to be spheres of constant density. The densities of rain drops, snow particles, and graupel particles are taken to be $\rho_r = \rho_l = 1000$ kg m$^{-3}$, $\rho_s = 100$ kg m$^{-3}$, and $\rho_g = 400$ kg m$^{-3}$, respectively. ($\rho_l$ refers to the density of liquid water.)
2. The size distribution (in terms of the actual diameter of the particles, rather than the melted diameter or the equivalent solid ice sphere diameter) is assumed to follow an exponential distribution of the form $N(D) = N_0 \exp(-\lambda D)$. The intercept parameter, $N_0$, is taken to be a constant value[1] of $8 \times 10^6$, $2 \times 10^7$, and $4 \times 10^6$ m$^{-4}$, for rain, snow, and graupel, respectively.

## 4. Calculation of $Z_e$ for rain drops

Assuming liquid water spheres and a size distribution as indicated in assumption 2 above, the sixth moment of the size distribution (equal to $Z$ and $Z_e$ for rain) is given by

$$Z_e = \Gamma(7) N_0 \lambda^{-7}, \tag{2}$$

where $\Gamma$ is the "gamma function", and $\Gamma(7) = 720$. This is the same as the expression given in Fovell and Ogura (1988), as applied to rain. The slope factor, $\lambda$, can be obtained from the model-predicted rainwater mixing ratio, $q_{\mathrm{ra}}$, using

$$\lambda = \left( \frac{\pi N_0 \rho_l}{\rho_a q_{\mathrm{ra}}} \right)^{1/4}, \tag{3}$$

where $\rho_a$ is the density of dry air.

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[1] More recent versions of MM5's mixed-phase bulk microphysical scheme use non-constant values of $N_0$ for rain and snow. See section 7 for further information.

## 5. Calculation of $Z_e$ for ice particles

It is known that within the Rayleigh scattering regime, the radar cross section of an irregular ice particle is the same as a solid ice sphere of equivalent mass (Marshall and Gunn 1952). While the assumed snow particles are spheres, they are not solid ice spheres. The diameter of the equivalent solid ice sphere is $D_{solid} = D_{snow}\left(\rho_s/\rho_i\right)^{1/3}$. Taking the sixth power of this relationship yields a factor of $\left(\rho_s/\rho_i\right)^2$ in the expression for equivalent reflectivity for the snow particles. Also, the reflective capacity of ice is less than that of water, by a factor equal to the ratio of the dielectric factor of ice ($|K|_i^2 = 0.176$) to that of liquid water ($|K|_l^2 = 0.930$), or 0.189. Considering these differences between ice and liquid water and the assumption number 1 above, the equivalent reflectivity factor of a population of snow particles is

$$Z_e = \Gamma(7)N_0\lambda^{-7}\left(\frac{\rho_s}{\rho_i}\right)^2\left(\frac{|K|_i^2}{|K|_l^2}\right), \tag{4}$$

where $\rho_i$ is the density of solid ice (taken to be 917 kg m$^{-3}$), and the value of $N_0$ is that for snow instead of rain. As in Smith (1984) and Fovell and Ogura (1988), (4) can be expanded and rearranged as

$$Z_e = \Gamma(7)N_0\lambda^{-7}\left(\frac{\rho_s}{\rho_l}\right)^2\left(\frac{\rho_l}{\rho_i}\right)^2\left(\frac{|K|_i^2}{|K|_l^2}\right) = \Gamma(7)N_0\lambda^{-7}\left(\frac{\rho_s}{\rho_l}\right)^2\alpha\,, \tag{5}$$

where $\alpha = \left(\rho_l/\rho_i\right)^2\left(|K|_i^2 / |K|_l^2\right) = 0.224$ is a constant[2] that does not depend on the assumed hydrometeor shape or density, only on the fact that the hydrometeor is frozen. Similar to rain, the slope factor, $\lambda$, can be obtained from the model-predicted snow water mixing ratio, $q_{\mathrm{sn}}$, using

$$\lambda = \left(\frac{\pi N_0\rho_s}{\rho_a q_{\mathrm{sn}}}\right)^{1/4}. \tag{6}$$

Analogous expressions to (5) and (6) can be derived for graupel particles, the only differences being that $N_0$ should be that for graupel instead of snow, $\rho_g$ should replace $\rho_s$, and $q_{\mathrm{gr}}$ should replace $q_{\mathrm{sn}}$.

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[2] The RIP algorithm currently uses the incorrect value of $\alpha = 0.213$. See section 7 for further information.

## 6. Calculation of total equivalent reflectivity factor in dBZ.

Since reflectivity, when *not* expressed in dBZ, is additive, the equivalent reflectivity value associated with each hydrometeor mixing ratio at a grid point can be calculated, and the values can be summed together to yield a total equivalent reflectivity factor. This quantity has MKS units of $\mathrm{m}^6\ \mathrm{m}^{-3}$, and should be multiplied by $10^{18}$ to convert it to the more common units of $\mathrm{mm}^6\ \mathrm{m}^{-3}$. The equivalent reflectivity in dBZ is then given by

$$Z_e \text{ (in dBZ)} = 10\log_{10}\left[Z_e \text{ (in mm}^6\text{ m}^{-3}\text{)}\right]. \tag{7}$$

This is the final quantity that is calculated and displayed in RIP's "dbz" algorithm.

## 7. Corrections and improvements for next version

The "dbz" algorithm in the current version of RIP (as of the date of this write-up) has a few aspects that could be improved and/or corrected. These changes are being considered for implementation in the next version of RIP, which will be released within a few weeks of this write-up.

a. In the "Reisner-2" mixed-phase bulk microphysical scheme currently employed in MM5, the intercept parameters for the rain and snow size distributions are actually not fixed at the constant values stated in section 3. Those constant values were used only in the earliest version of the scheme (ca. 1996). The specification of the intercept parameter has been an evolving aspect of the microphysical scheme. The intercept parameter for snow was changed from a constant value to a mixing ratio-dependent value as described in Reisner et al. (1998), and then was changed to a temperature-dependent value as described in Thompson et al. (2004). Warmer temperatures yield lower $N_{0s}$ to account for the effects of enhanced aggregation. The intercept parameter for rain in the MM5's most recent mixed-phase scheme is diagnosed from the rain mixing ratio to reproduce a more realistic "drizzle-like" size distribution at low mixing ratios. If the "dbz" algorithm is applied to MM5 model output, these diagnosed intercepts for rain and snow should be employed in the "dbz" algorithm, to be more consistent with the MM5's bulk mixed-phase scheme. However, other versions of the "dbz" algorithm should also be developed for output from other models whose microphysical schemes have different assumptions about particle size distributions.

b. The current version of the algorithm uses $\alpha = 0.213$, following Fovell and Ogura (1988) and others. This value is erroneous as discussed by Fovell (2003) and should be replaced with the correct value of $\alpha = 0.224$, as given by Smith (1984). Note, however, that this correction will result in only a 0.2 dBZ increase in simulated reflectivity factor, and only for the contributions from snow and graupel.

c. A common feature of stratiform precipitation is a "brightband" of enhanced reflectivity at and below the melting level. This is thought to be caused by two effects: the creation of a liquid "skin" on melting snow flakes, which increases their dielectric factor to the liquid value while the snow is still falling at the slower terminal velocity of snow flakes; and the enhanced aggregation that occurs among these "sticky" snow flakes, to which reflectivity (with its $D^6$ dependence) is highly sensitive. While an accounting for the enhanced aggregation would be complicated, it would be easy to implement the enhanced dielectric factor effect, by simply setting the dielectric ratio $\left(|K|_i^2 / |K|_l^2\right)$ to 1 in the calculation of equivalent reflectivities for snow and graupel in regions where $T > 0$ ºC.

d. The current "dbz" algorithm accounts only for Rayleigh scattering. A more complete algorithm would also account for Mie scattering effects, and others have done this. However, since Rayleigh scattering is the dominant scattering regime for weather radar, especially for S-band (10-cm) radars like the National Weather Service's WSR-88D radars, the more complete approach makes only a few dBZ of difference at the most, and only for the strongest radar echoes where large particles (like hailstones) are present. Therefore, I do not anticipate adding Mie scattering to the algorithm.

## References

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