Understanding the importance of microphysics and macrophysics for warm rain in marine low clouds: Part II. Heuristic models of rain formation.

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ABSTRACT

Two simple heuristic model formulations for warm rain formation are introduced and their behavior explored. The first, which is primarily aimed at representing warm rain formation in shallow convective clouds, is a continuous collection model that uses an assumed cloud droplet size distribution consistent with observations as the source of embryonic drizzle drops that are then allowed to fall through the (assumed constant) cloud accreting cloud droplets. The second, which is applicable to steady-state precipitation formation in stratocumulus, is a simple two-moment autoconversion and accretion model in which cloud liquid water is removed by drizzle formation and replenished on an externally-specified timescale that reflects the efficacy of turbulent overturning that characterizes stratocumulus.

The models' behavior is shown to be broadly consistent with observations from the A-Train constellation of satellites, allowing us to explore reasons for changing model sensitivity to microphysical and macrophysical cloud properties. The models are consistent with one another, and with the observations, in that they demonstrate that the sensitivity of rain rate to cloud droplet concentration N_d (which here represents microphysical influence) is greatest for weakly precipitating clouds, i.e. for low cloud liquid water path and/or high N_d . For the steady-state model, microphysical sensitivity is shown to strongly decrease with the ratio of replenishment to drizzle timescales. Thus, rain from strongly drizzling and/or weakly replenished clouds shows low sensitivity to microphysics. This is essentially because most precipitation in these clouds is forming via accretion rather than autoconversion. For the continuous-collection model, as cloud liquid water content increases, the precipitation rate becomes more strongly controlled by the availability of cloud liquid water than by the initial embryo size or by the cloud droplet size. The models help to explain why warm rain in marine stratocumulus clouds is sensitive to N_d but why precipitation from thicker cumulus clouds appears to be less so.

1. Introduction

In part I of this study (Kubar et al. 2009), observations from the A-Train satellites were used to determine characteristics of warm rain formation in marine low clouds, and to investigate the cloud macrophysical and microphysical factors that influence it. In most regions, greater drizzle intensity (higher radar reflectivity) is associated with significant increases in cloud top height and cloud liquid water path LWP but with decreases in cloud droplet concentration N_d that are more modest. This is particularly true for regions over the remote oceans that are relatively pristine. In polluted regions off the East Asian coast and over the Gulf of Mexico, higher liquid water contents are required to give the same drizzle intensity as clouds over pristine regions suggesting a suppression of precipitation due to higher cloud droplet concentrations.

Studies of drizzle formation in marine stratocumulus show a significant role for N_d in limiting drizzle (Pawlowska and Brenguier 2003; Comstock et al. 2004; Van Zanten et al. 2005; Wood 2005a; Geoffroy et al. 2008). In contrast, studies in the more strongly precipitating trade cumulus regime do not find a clear link between N_d and precipitation rate (Nuijens et al. 2009). The lack of microphysical sensitivity in shallow convection is largely driven by the increasing dominance of the accretion process over auto-conversion as precipitation rates increase (Stevens and Seifert 2008). This is because the accretion rate is almost independent of N_d (it depends only on the product of the cloud and rain mixing ratios) whereas autoconversion is strongly sensitive to N_d (Beheng 1994; Khairoutdinov and Kogan 2000; Liu and Daum 2004; Wood 2005b).

Here, we introduce two simple heuristic models to attempt to understand the behavior seen in the observations introduced in Part I of this study. We find that increasing importance of accretion as the precipitation rate increases is evident in simple models and largely explains why we would expect the microphysical sensitivity of precipitation in warm rain to be a decreasing function of the precipitation efficiency.

2. Model formulations

a. Continuous collection (CC) model

We derive a simple continuous collection model that determines the precipitation rate at the base of an idealized warm cloud resulting from drops falling and accreting cloud water. It is primarily aimed at reproducing the instantaneous precipitation falling from a cloud that is assumed not to change during the course of the precipitation event. The model inputs are the cloud thickness (or alternatively its liquid water path) and the cloud top effective radius (or alternatively the cloud droplet concentration).

The following assumptions are made:

1. *Assumed macrophysical cloud structure:* The cloud consists of a layer with a constant specified cloud droplet concentration N_d and a cloud liquid water content ρq_l that increases with height above cloud base z at a specified rate $\rho dq_l/dz = \Gamma$, where q_l is the cloud water mixing ratio and ρ is the air density. The liquid water gradient Γ is given by $\Gamma = f_{ad}\Gamma_{ad}$ where Γ_{ad} is the thermodynamically determined increase for an adiabatic parcel ascent and f_{ad} is the adiabaticity factor. We parameterize $f_{ad} = z_0/(z_0 + z)$ where z_0 is a scaling parameter, set to 500 m, which matches very well with liquid water content observations in warm marine clouds (Rangno and Hobbs 2005) with depths from 1-4 km. Thus shallow clouds tend to be more close to adiabatic than deeper ones consistent with arguments for entraining plumes.

2. *Assumed microphysical cloud structure:* The cloud droplet size distribution $n(r)$ is assumed to be a Gamma distribution (Austin et al. 1995; Wood 2000), with a spectral width determined as a function of the mean volume radius $r_v = (3\rho q_l/4\pi\rho_w N)^{1/3}$ using the parameterization of Wood (2000) and accounts implicitly for the narrowing of the size distribution due to condensational growth and its broadening (in a fractional sense) due to increased CCN concentration.

3. *Precipitation embryos*: At the cloud top, a small subset of the largest cloud droplets (specified number concentration N_D) are considered to be precipitation embryos that subsequently are allowed to fall through the cloud layer and grow by coalescing with smaller cloud droplets. Here we specify N_D , which is a free parameter, in accordance with observed concentrations of precipitation drops in warm precipitating clouds and use this to determine the minimum size r_- of the drizzle embryos using

$$\int_{r_-}^{\infty} n(r) dr = N_D \quad (1)$$

We then use N_D and r_- and the assumed cloud droplet size distribution to determine the mean mass-weighted radius of the embryos R_{emb} , i.e.

$$R_{emb} = \left[\frac{1}{N_D} \int_{r_-}^{\infty} r^3 n(r) dr \right]^{1/3} \quad (2)$$

An alternative would be to specify r_- as a free parameter, and the use (1) to determine N_D . We experimented with this approach and found that the salient findings were not markedly different. For simplicity, we do not report further on these experiments.

4. *Coalescence growth of drizzle drops*: The drizzle embryos, which are represented by a single embryo with an initial radius R_{emb} then fall through the depth of the cloud continuously collecting cloud droplets and growing in radius at a rate that is approximately the product of the cloud liquid water content, the collector drop fall speed, and a collection efficiency (Rogers and Yau 1989). The cloud droplets collected are assumed to have a size equal to r_v at that level. A more complete description of the radius growth rate (Rogers and Yau 1989) of the falling drizzle drop R with respect to height is

$$\frac{dR}{dz} = - \left(1 + \frac{r_v}{R} \right)^2 \left(1 - \frac{v_T(r_v)}{v_T(R)} \right) \frac{\rho_{ql} E(R, r_v)}{4\rho_w} \quad (3)$$

where v_T is the terminal velocity of a falling drop, $E(R, r_v)$ is the collection efficiency of the falling drizzle drop and the collectee cloud droplets, and ρ_w is the density of liquid water. Terminal velocities are determined using the relations given in Pruppacher and Klett (1997). Collection efficiencies are taken from Hall (1980). The negative sign in (3) indicates that the drizzle drops grow downwards. The drops are assumed to be falling in still air.

5. *Determining bulk drizzle characteristics*: At any level in cloud ($0 < z < h$) the drizzle liquid water mixing ratio $q_{l,D}$ and precipitation rate P and Rayleigh radar reflectivity factor Z are determined using R :

$$q_{l,D} = \frac{4\pi\rho_w}{3\rho} N_D R^3 \quad (4)$$

$$P = \frac{4\pi\rho_w}{3} \alpha_T N_D R^{3+\beta} \quad (5)$$

$$Z = 2^6 N_D R^6 \quad (6)$$

Here, we use the approximate formulation for the terminal fall speed $v_T = \alpha_T R^\beta$ with $\alpha = 2.2 \times 10^5 \text{ m}^{-0.4} \text{ s}^{-1}$ and $\beta = 1.4$, values taken from (Comstock et al. 2004).

1) CC MODEL - FREE PARAMETERS

The key free parameters of the CC model, as shown in Table 1 are the cloud thickness h (or equivalently LWP as it is uniquely related to h) and the cloud droplet concentration N_d . Experimentation shows

relatively weak sensitivity to the choice of the liquid water adiabaticity scaling parameter z_0 . We use a constant value of $\Gamma_{ad} = 2 \times 10^{-6} \text{ kg m}^{-4}$.

The cloud top effective radius r_e^+ , which can be estimated from satellite measurements, is determined uniquely from the mean volume radius at the top of the cloud r_v^+ which is a function of LWP and N_d only.

b. Steady-state autoconversion/accretion model

The second model we introduce is a highly simplified model of precipitation formation in a stratiform layer cloud in which there exists (in steady-state) a balance between a loss of cloud water through collision-coalescence and its replenishment by assumed turbulent motions which drive the cloud back towards an adiabatic layer. It is primarily designed as a heuristic model to reproduce the equilibrium behavior of precipitation stratiform boundary layer clouds. The model uses a two moment bulk microphysical formulation, with prognostic equations for the cloud water, precipitation water, and the precipitation drop concentration, and the following assumptions are made:

1. *Prognostic bulk microphysical equations:* The cloud consists of a single vertically-homogeneous layer of thickness h . The cloud state is characterized by a cloud droplet concentration N_d which is an external, time-invariant parameter, and a cloud liquid water mixing ratio q_l which evolves in time. The precipitation is characterized by a rain mass mixing ratio q_r and a rain drop concentration N_D both of which evolve. The following equations describe the evolution of the three prognostic variables:

$$\rho \frac{dq_l}{dt} = \rho \frac{(q_{ad} - q_l)}{\tau_{rep}} - A_c - K_c \quad (7)$$

$$\rho \frac{dq_r}{dt} = A_c + K_c - S_q \quad (8)$$

$$\frac{dN_D}{dt} = \frac{A_c}{m_{emb}} - S_N. \quad (9)$$

Here q_{ad} is the adiabatic mean liquid water mixing ratio for the layer (determined as $\frac{1}{2}\Gamma_{ad}h$, see Section 2a above), to which q_l relaxes with a replenishment timescale τ_{rep} . The autoconversion rate A_c is a specified function of the cloud state q_l, N_d (see below). The accretion rate is specified as $K_c = \gamma \rho^2 q_l q_r$, with $\gamma = 4.7 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1}$ as in the formulation of Tripoli and Cotton (1980). Observations suggest that diversity is small across different accretion rate formulations (Wood 2005b), and we find (not shown) that our findings are barely altered by using a different formulation.

The sedimentation rates for rain water S_q and rain number S_N are specified as $S_q = 2\rho q_r v_{T,q}/h$ and $S_N = 2N_D v_{T,N}/h$ respectively, where $v_{T,q}$ and $v_{T,N}$ are the fall speeds for the 3rd and zeroth moment of the rain size distribution each of which is specified as a linear function of the rain drop volume radius $r_{v,D} = [3\rho q_r / (4\pi\rho_w N_D)]^{\frac{1}{3}}$ using the parameterization of Khairoutdinov and Kogan (2000). Finally, m_{emb} is the mass of a drizzle drop embryo formed by autoconversion, which we assume to have a radius of 22 μm . The results are not highly sensitive to the exact choice of this radius.

2. *Sensitivity to autoconversion:* Available expressions for the autoconversion rate differ markedly in their sensitivities to q_l and N_d (Wood 2005b), and so we investigate the sensitivity of our findings to these differences by using a number of different autoconversion parameterizations. We use the formulations of Khairoutdinov and Kogan (2000) (KK), Liu and Daum (2004) as modified by Wood (2005b) (here denoted as LD), Beheng (1994) (BEH), and Seifert and Beheng (2001) (SB).

3. *Steady state solutions:* The model is run with $q_l(t=0) = 0$ until a steady state has been reached.

4. *Determining bulk drizzle characteristics:* The model precipitation rate is $P = \rho q_r v_{T,q}$. The model estimates a Rayleigh radar reflectivity factor due to precipitation Z from the two moments q_r and N_D

by assuming an exponential size distribution truncated at the assumed threshold for cloud and drizzle drop ($r = 20\mu\text{m}$).

1) STEADY STATE MODEL - FREE PARAMETERS

The free parameters of the steady-state autoconversion and accretion model are the cloud thickness h , the cloud droplet concentration N_d , and the cloud water replenishment timescale τ_{rep} (see Table 1 for values used here). We use a constant value of $\Gamma_{ad} = 2 \times 10^{-6} \text{ kg m}^{-4}$.

3. Model behavior

a. Sensitivity of precipitation rate to LWP and N_d

Before attempting to address the question of how faithfully our two models are able to capture the salient features of the A-Train observations, it is useful to explore some of the essential model behavior.

Figure 1 shows the cloud base precipitation rate P_{CB} as a function of the cloud LWP and the cloud droplet concentration N_d from the CC and SS models (the latter with the Khairoutdinov and Kogan autoconversion parameterization). Both models show that the highest precipitation rates are associated with clouds with high LWP and also with low N_d , and both show similar regions of phase space where P_{CB} is effectively zero. The models are in the best agreement for $N_d < 100 \text{ cm}^{-3}$. At higher N_d there is a significantly stronger dependence of P_{CB} upon N_d in the CC model than in the SS model. The Khairoutdinov and Kogan autoconversion parameterization was derived from bin-resolved large eddy simulation for N_d typical of clean marine conditions (Khairoutdinov and Kogan 2000), with almost all N_d values below 120 cm^{-3} and so we might not expect it to perform well at high N_d . However, a similar discrepancies exist between the CC and SS models at $N_d > 100 \text{ cm}^{-3}$ for the other autoconversion parameterization schemes used in the SS model. Thus, the differences are not likely related to subtleties in the autoconversion parameterizations used.

Both models show a steepening of the P_{CB} isolines (isohyets) as P_{CB} increases (Fig. 1), reflecting an increase in the relative dependence upon LWP compared with N_d at large P_{CB} . We can understand this behavior in the CC using a minimal analytical CC model (see the Appendix) which demonstrates that as LWP increases the key process limiting the rainrate becomes the availability of cloud liquid water to accrete onto growing raindrops rather than the initial size of the precipitation embryos or their efficiency as collector drops to accrete cloud liquid water. However, at high droplet concentrations typical of polluted conditions (N_d of a few hundred per cm^3 or more), the initial embryo size can still be an important limiter of precipitation formation even at LWP typical of a few hundred g m^{-2} . Insofar as the CC model is a reasonable replicator of reality, these results have important implications for the sensitivity of warm rain formation to cloud microphysics.

It should be noted however, that the CC model precipitation isolines are much more tightly packed for high LWP and high N_d than those from the SS model. This behavior is more or less reproduced by the minimal CC model (see Fig. 12 in the Appendix) and is caused by the extreme sensitivity of the collection efficiency to droplet size for droplets of around $10 \mu\text{m}$ and smaller (see Fig. 11). The SS model does not reproduce this behavior because its bulk autoconversion formulations are not particularly designed to capture this sensitivity.

In the next section we attempt to constrain the model using observational data from the A-Train satellites.

b. Sensitivity to model parameterizations

1) CC MODEL

The CC model is sensitive to the number concentration N_D of precipitation embryos (see model assumption 3 in Section 2a) which is set equal to 100 liter^{-1} , a number towards the upper end of observed concentrations of drops with radii larger than $20 \mu\text{m}$ in precipitating warm clouds (Hudson and Svensson

—1995—; Comstock et al. 2004; Wood 2005a). We find that a doubling of N_D results in an increase of 50-80% in precipitation rate. The model precipitation rate increases somewhat more slowly than N_D itself because a higher N_D means a smaller mass weighted embryo radius R_{emb} (see model assumption 3 in Section 2a). However, there is little doubt that the need to specify N_D remains a major limitation of the CC model.

The CC model precipitation rate is also somewhat sensitive to z_0 , but z_0 primarily controls the relationship between the cloud depth and the cloud LWP. For a given cloud thickness, there is a strong dependency of P_{CB} upon z_0 but when expressed as a function of LWP, there is a relatively weak dependence on z_0 . Increases of only 5-20% in P_{CB} are found for a given $[LWP, N_d]$ pair as z_0 increases from 250 m to 1000 m.

2) SS MODEL

The SS model results display some sensitivity to the autoconversion parameterization used. Figure 2 shows that the greatest fractional sensitivity of precipitation rate to autoconversion parameterization is generally at low values of P_{CB} , i.e. at high LWP and low N_d . For example, at $LWP = 100 \text{ g m}^{-2}$ and $N_d = 50 \text{ cm}^{-3}$, P_{CB} values are 0.05, 0.15, 0.13, and 0.5 mm day^{-1} (i.e. an order of magnitude spread) for the four parameterizations (LD, KK, BEH, SB) respectively, while at $LWP = 400 \text{ g m}^{-2}$ and $N_d = 20 \text{ cm}^{-3}$, P_{CB} values are 13, 31, 45, and 55 mm day^{-1} respectively. For reference, the CC model rates are 0.2 and 38 mm day^{-1} respectively for the low and high LWP cases.

Thus, there is a somewhat weaker sensitivity of P_{CB} to autoconversion parameterization at high P_{CB} . However, more striking is that the sensitivity of P_{CB} to changes in LWP and N_d is much less dependent upon the autoconversion parameterization at high P_{CB} . This can be seen by inspection of the P_{CB} isolines in Fig. 2, which are less autoconversion-dependent to the lower right of the panels.

In other words, although the autoconversion impacts the precise value of P_{CB} , the role of the autoconversion parameterization in determining the sensitivity of P_{CB} to changes in e.g. aerosols, is diminished for strongly precipitating clouds. We return to this in the discussion.

The SS model P_{CB} also displays some sensitivity to the cloud liquid water replenishment timescale τ_{rep} , and Fig. 3 shows that sensitivity to τ_{rep} is strongest at high P_{CB} . This is because at low P_{CB} the clouds are close to adiabatic because sedimentation is inefficient at removing cloud water. As the precipitation efficiency increases (i.e. the timescale for precipitation removal becomes comparable with τ_{rep}), increasing the replenishment rate of cloud liquid water (decreasing τ_{rep}) permits a larger total rate of conversion of cloud to rain and a larger P_{CB} . However, while Fig. 3 shows that there is sensitivity to τ_{rep} , the sensitivities to LWP and N_d tend to be dominant. The results for the other autoconversion parameterizations are qualitatively very similar.

We should note that the SS model does not attempt to parameterize the effect of turbulence on recycling of drizzle drops themselves, which is known to be important for the formation of the largest drizzle drops in precipitating stratocumulus (Nicholls 1989; Baker 1993; Austin et al. 1995). This is difficult to achieve within the model framework used here.

4. Observations

In this study we use observations of radar reflectivity from the CloudSat satellite and visible/near infrared estimates of cloud liquid water path and collocated cloud droplet concentration from MODIS on NASA's Aqua satellite. Extensive details regarding the observations and their limitations can be found in Part I of this manuscript (Kubar et al. 2009).

In this part of the study we attempt to reproduce, using the CC and SS models the dependency of the column maximum radar reflectivity upon the cloud LWP and N_d . We use the eight different regions over the subtropical and tropical Pacific Ocean and Gulf of Mexico as defined in Kubar et al. (2009) (see Table 2 here and also their Fig. 5), together with a region that covers the entire oceanic region (30°S - 30°N , 100°E - 70°W).

For each region joint pdfs of LWP and effective droplet concentration¹ (N_{eff}) are constructed from the MODIS retrievals for all the matched CloudSat/MODIS data with detectable radar reflectivity and retrieved cloud properties. A selection of statistical properties is shown in Table 2. In Part I we demonstrated that the radar reflectivity (which for reflectivities greater than -15 dBZ is inferred to be precipitation) scales with both LWP and N_{eff} , which we refer to here as our macrophysical and microphysical cloud properties. Here, we will ask how faithfully our heuristic models can reproduce the general characteristics of the radar reflectivity as a function of LWP and N_{eff} .

5. Comparing the models with observations

a. Treatment of CC model output for comparison with observations

For the CC model the precipitation rate and reflectivity factor due to precipitation maximize at the cloud base. The radar reflectivity factor at the cloud base Z_{cb} is corrected for Mie scattering at 94 GHz (the wavelength of the CloudSat radar) using the approach of Comstock et al. (2004). For a radar reflectivity factor of 7.5 dBZ (which corresponds to a rain rate of approximately 2 mm hr⁻¹), the Mie correction is approximately 2.5 dBZ and increases by approximately 1 dBZ for each 2 dBZ increase in Z_{cb} . Because few of the shallow clouds in this study produce rain significantly heavier than this, the attenuation correction for warm rain does not have a major impact.

To further facilitate comparison with CloudSat the Mie-corrected model reflectivity is then corrected for two-way attenuation at 94 GHz given the cloud liquid water path using a two-way attenuation coefficient of 8.4 dBZ (1000 g m⁻²)⁻¹ (Lhermitte 1990). This correction becomes significant for high LWP, but for most LWP values here (typically up to a few hundred g m⁻²) it is relatively modest. Both the Mie scattering and attenuation corrections affect the statistics for the subset of clouds that is precipitating most strongly. The corrected model reflectivity is given the symbol Z_{model} .

The cloud particles that in the model are the initial source of the precipitation can also scatter and frequently dominate the reflectivity signature when the precipitation rates are lower than a few tenths of a mm day⁻¹. The assumed gamma distribution is used to determine the cloud reflectivity, which is largest at cloud top. Because of this and because Mie scattering is not important for cloud drops, the reflectivity from cloud drops does not require corrections. Finally, the model's column maximum reflectivity is determined as the maximum of that due to cloud and precipitation.

b. Treatment of steady-state model data

The SS model radar reflectivity due to precipitation (which is assumed to maximize at cloud base) is corrected for Mie scattering and attenuation as a function of LWP in the same way as for the CC model. The mean volume radius of the drizzle drops $r_{v,D}$ is used to determine a Mie scattering correction, and the LWP is used to determine the attenuation.

The cloud contribution to the reflectivity (which is assumed to maximize at cloud top despite the model lacking an explicit vertical dimension), is determined by assuming a gamma distribution for the cloud droplet size distribution, with inputs q_l , N_d , and a parameterized spectral width in exactly the same manner as for the CC model (see assumption 2 in Section 2a above). The reflectivity from cloud droplet does not require correction for Mie scattering or attenuation.

c. Comparison with A-Train data

The CC model is run for $10 < LWP < 1000$ g m⁻² and $10 < N < 1000$ cm⁻³. The SS model is run for a $100 < h < 3000$ m which ensures LWP in the range of 10-1000 g m⁻², and $10 < N < 1000$ cm⁻³.

We then use the observationally-derived joint pdfs of LWP and N_d to determine statistics of the

¹The effective droplet concentration N_{eff} is retrieved from the MODIS estimates of cloud optical thickness and cloud top effective radius under the assumption that the cloud liquid water content follows an adiabatic vertical profile. See Part I for further details.

population of clouds in each of our eight focus regions. The statistics we determine from the observations and model are:

- (a) the fraction of observed clouds that are precipitating (i.e. have a column maximum corrected reflectivity $Z_{model} > -15$ dBZ);
- (b) the fraction of clouds with moderate drizzle (corrected reflectivity $Z_{model} > 0$ dBZ);
- (c) the fraction of clouds with heavy drizzle ($Z_{model} > 7.5$ dBZ);
- (d) the median reflectivity of all clouds;
- (e) the median reflectivity of precipitating ($Z_{model} > -15$ dBZ) clouds. For the observations, these values are presented by region in Table 1 of Part I of this study.

Given the $Z - R$ relationship of Comstock et al. (2004), and taking into account the attenuation/Mie corrections appropriate for 94 GHz, a cloud base reflectivity of -15 dBZ corresponds to approximately 0.25-0.5 mm day⁻¹, while 0 dBZ corresponds to ~2-5 mm day⁻¹, and 7.5 dBZ corresponds to ~20-40 mm day⁻¹, but we should caution that the attenuation and Mie corrections severely upset the uniqueness of the relationship between reflectivity at 94 GHz and rain rate.

6. Comparison results

a. Comparison in the LWP, N_{eff} plane

Figure 4 shows the column maximum 94 GHz radar reflectivity as a function of LWP and N_d from the continuous collection model and for the entire set of A-Train observations within 30°S-30°N, 100°E-70°W. In general, both models reproduce critical aspects of the observations with some fidelity. Especially good is the ability to reproduce the threshold between precipitating and nonprecipitating clouds (i.e. the -15 dBZ contour), although the CC model is less good at reproducing the observed dBZ structure at high values of LWP and N_{eff} than is the SS model. Both models quite successfully capture the broad change in contour slope as the clouds transition from nonprecipitating to precipitating.

The CC model appears to be more successful than the SS model at capturing the weakening microphysical dependency at high P_{CB} that is seen in the observations and reproduced well by the CC model. However, a comparison of Figures 1 and 4 suggests that this reflects a different relationship between P_{CB} and Z in the SS model than in the CC model, because P_{CB} isolines in the SS model do become more vertical at increased P_{CB} even though the reflectivity isolines do not. However, the P_{CB} isolines in the CC model at high LWP and low N_d are still steeper than the SS model for all autoconversion parameterizations other than SB (see Fig. 2).

b. Comparison of precipitation characteristics by region

For each of the eight regions described in Table 2 we use the observed joint pdf of LWP and N_{eff} to produce model estimates of the metrics introduced in Section 5c. These are compared with the observed values in Figures 5 and 6. In general, both models successfully reproduce the fraction of observed/screened² clouds that are precipitating (those with reflectivities > -15 dBZ), and also those with dBZ > 0 (moderate drizzle). Thus, to a good degree, both models can determine important characteristics about the distribution of light and moderate drizzle for warm clouds given their LWP and cloud droplet concentration.

However, the models cannot accurately reproduce the fraction of clouds with heavy drizzle (dBZ > 7.5), although the CC model displays some skill. This is perhaps not surprising for the SS model given that it is primarily designed to simulate precipitation in overturning stratocumulus clouds which rarely exhibit precipitation rates greater than 20 mm day⁻¹ (Comstock et al. 2004).

It is interesting that the observations show good relationships between the fraction with moderate

²We should note that the observed drizzling fraction reported here should not be confused with the fraction of *all* clouds that are drizzling, but is the fraction of clouds selected as being detectable by both CloudSat and MODIS, and being appropriately screened as being optically thick and relatively homogeneous as detailed in Kubar et al. (2009).

drizzle and the fraction with any drizzle, and between the fraction with heavy drizzle and that with moderate drizzle (Fig. 7) for the observations and for the CC model. The observed relationships are reproduced fairly well assuming that the distribution of reflectivity in each region is lognormal with a fixed geometrical standard deviation of 14 dBZ (and a variable mean). The CC model is able to capture the essence of these relationships with some skill.

The models also perform reasonably well in reproducing the median reflectivities for the different regions (Fig. 6). However, both models underestimate the precipitating fraction (> -15 dBZ) and the median reflectivity (especially the CC model) for the Asian coast and Gulf of Mexico regions, which are the two regions with the highest median N_{eff} and LWP. For the CC model, this could have been anticipated given the relatively poor comparison of the model and observed reflectivity structure in the (LWP, N_{eff}) plane for high LWP and N_{eff} (see previous section and Fig. 4). Figure 13 reminds us that this is the region of phase space where the characteristic droplet radii around $10\text{ }\mu\text{m}$ or smaller, the approximate size at which the collection efficiency changes rapidly with droplet size (e.g. Rogers and Yau 1989). This certainly explains the strong sensitivity of P_{CB} to droplet size in this region for the CC model. The SS model also underestimates the median reflectivity for the ITCZ/SPCZ region which may reflect its unsuitability for predicting precipitation in strongly drizzling, cumuliform clouds.

7. Discussion

a. Sensitivity of warm rain to microphysics and macrophysics

We have seen that the observed distribution of reflectivity can be reproduced with some skill using the simple heuristic models presented here. It thus gives us some confidence that we can use the model to infer aspects of the sensitivity of warm rain to changes in macrophysical and microphysical cloud properties.

In Fig. 8 we show, for the CC model, the sensitivity of the distribution of reflectivity in each region to an across-the-board increase of 50% in LWP and a 50% *decrease* in N_{eff} . The behavior of the SS model (not shown) is very similar. Increased LWP results in a shift to the right of the reflectivity pdfs by a substantial amount (roughly 4-5 dBZ, equivalent to around a factor of 2.5-3 increase in reflectivity), whereas the reduction in N_{eff} induces significantly more modest increases in reflectivity. Further, the microphysical sensitivity decreases as the reflectivity increases (from 1-2 dBZ around 0 dBZ to less than 1 dBZ at 5-10 dBZ), consistent with the behavior seen in Fig. 4, and explained theoretically (see Appendix) as being due to a shift to an accretion-limited regime at high reflectivity. Note that a shift of $10 \log_{10}(1.5) = 1.76$ dBZ is equivalent to a 50% increase in reflectivity.

To confirm that these results are not strongly dependent upon our assumptions about the reflectivity-rain rate relationship, we show a similar plot for the modeled cloud base precipitation rate P_{CB} distributions for the NE Pacific region (Fig. 10). At the high precipitation rates, the sensitivity of P_{CB} to changes in LWP is much greater than to changes in N_{eff} . This behavior is repeated for the other regions (not shown).

These model results, when taken together with the observed tendency for the reflectivity to become more strongly dependent upon LWP as the reflectivity increases (Fig. 4), suggests a markedly diminished ability for cloud microphysics to influence precipitation rates in clouds that are accretion-dominated. Moreover, the ability to unequivocally attribute observed variability in precipitation rates to microphysical changes will therefore be more difficult for clouds with higher precipitation rates. This finding largely explains why, although there is strong evidence for N_d -limited precipitation in drizzling stratocumulus (Pawlowska and Brenguier 2003; Comstock et al. 2004; Van Zanten et al. 2005; Wood 2005a; Geoffroy et al. 2008), there is no such evidence for substantial N_d -limitation of precipitation in clouds with markedly greater liquid water paths than those found in stratocumulus (Levin and Cotton 2008). Existing observational studies supporting such microphysical impacts are largely flawed in part because they have inferred precipitation occurrence from measures such as cloud top effective radius (see Part I of this study) that have little direct connection to precipitation rate.

b. Steady state precipitation

For the steady state model, in which a balance is reached between loss of cloud water by conversion to drizzle and replenishment via turbulent updrafts (timescale τ_{rep} , see Section 2b), we can define an additional timescale τ_{driz} for the conversion of cloud to drizzle,

$$\tau_{driz} = \frac{\rho q_c}{A_c + K_c} \quad (10)$$

A remarkable behavior of the steady state model is that the microphysical susceptibility of the cloud base precipitation rate $S = d\ln P_{CB}/d\ln N_d$ is very strongly related to the ratio of the replenishment to depletion timescales τ_{rep}/τ_{driz} . Further, if we normalize S by the sensitivity of the autoconversion parameterization to changes in N_d , i.e. $\beta = d\ln A_c/d\ln N_d$, then we find that S/β is almost independent of the autoconversion parameterization.

What this tells us is that clouds in steady state with strong drizzle (low τ_{driz}) or slow replenishment (high τ_{rep}) have a lower sensitivity to N_d than those with weak drizzle and rapid replenishment. Observations in stratocumulus clouds (Wood 2005a) suggest that replenishment timescales for liquid water in stratocumulus may be a few times the eddy turnover timescale, or around 1-2 hours. This is consistent with the lifetimes of mesoscale drizzle cells that frequently dominate the dynamics of these clouds (Comstock et al. 2007). Given that typical liquid water contents in stratocumuli are $\sim 0.5 \text{ g kg}^{-1}$, and that $A_c + K_c$ is of the order of $5 \times 10^{-9} - 5 \times 10^{-8} \text{ kg m}^{-3} \text{ s}^{-1}$ (for precipitation rates of the order of $0.5 - 5 \text{ mm day}^{-1}$), this would put τ_{driz} in the range $10^4 - 10^5 \text{ s}$, and S/β in the range 0.2-0.8. Even drizzling stratocumulus clouds may therefore exhibit a sensitivity to N_d that is substantially weaker than the sensitivity of autoconversion itself.

The reasons for this behavior are again a result of the increased importance of accretion as the precipitation becomes more efficient at depleting cloud liquid water. In this way the behavior of the SS model is generally consistent with the behavior of the CC model.

8. Conclusions

In this study, we have attempted to reproduce some of the salient observational results detailed in Part I of this study (Kubar et al. 2009). These observations, for which important new data from the CloudSat satellite is combined with visible/near IR data from MODIS, demonstrate that the radar reflectivity in precipitating low clouds is influenced both by variability in macrophysical quantities like cloud liquid water path LWP, and by microphysical quantities such as the estimated cloud droplet concentration N_{eff} . The observations demonstrate that for clouds with higher reflectivities, the reflectivity is more sensitive to LWP than to N_{eff} . This general tendency is reproduced well by a continuous collection (CC) model of precipitation formation, and can be explained by the increasing importance of accretion in controlling the precipitation amount when the precipitation rate is larger than a few mm day^{-1} .

The general behavior of the observations is replicated using a minimal analytical CC model that aids understanding. Because accretion is largely controlled by the availability of liquid water and is not strongly limited by reduced collection efficiency, this leads to a weakened dependence of precipitation upon N_{eff} at high LWP in the accretion-dominated regime. Quantitatively similar behavior is obtained in a bulk microphysical steady state precipitation model, but there is strong quantitative sensitivity to the choice of autoconversion parameterization.

Finally, the observations suggest that the influence of anthropogenic aerosols (through their ability to act as CCN and influence the cloud droplet concentration) upon warm rain is likely to be a strong function of the cloud microphysical and macrophysical state of the clouds into which the aerosols are being ingested. In other words, the susceptibility of warm rain to microphysical changes will likely depend upon the type of clouds, and therefore the meteorological regime. It will be useful in future studies to attempt to examine this susceptibility as a function of the liquid water path and the cloud droplet concentration.

Appendix: Minimal CC model

The form of the relationship between LWP , N_d and P_{CB} in the CC model is strongly controlled by the dependence of the collection efficiency upon the collector drop size. To demonstrate this, a minimal model is constructed that simplifies the warm rain model into an analytical form.

We begin with (3) and retain the R dependence only through the collection efficiency, i.e. $E(R, r_v)$. This is reasonable since $\eta \equiv (1 + r_v/R)^2[1 - v_T(r_v)/v_T(R)]$ does not vary strongly with r_v/R , whereas the collection efficiency drops dramatically for small droplets. We set $\eta = 1.2$, and assume a constant value of r_v through the cloud layer. We choose r_v as the liquid water weighted mean value of r_v for the cloud layer. It can be shown that this is 6/7 times the value at the cloud top r_v^+ .

We then integrate (3) from cloud top to cloud base

$$\int_{R_{emb}}^{R_{CB}} E^{-1}(R, r_v) dR = \frac{\eta LWP}{4\rho_w} \quad (11)$$

We then parameterize $E^{-1}(R, r)$, which can be thought of as a collection inhibition factor, as the product of a function depending upon the collector drop radius R and a function depending upon the collected drop radius r_v , i.e.

$$E^{-1}(R, r) = Q_r Q_R \approx [1 + (a/R)^5] [1 + (b/r)^4] \quad (12)$$

with $a = 30.6 \mu\text{m}$ and $b = 6.27 \mu\text{m}$. Fig. 11 shows $E^{-1}(R, r)$ from Hall (1980) and from the (12). The parameterization captures the inhibition of coalescence for small values of the collected drop and particularly the collector drop. With the parameterization (11) becomes

$$\left[R - \frac{a^5}{4} R^{-4} \right]_{R_{emb}}^{R_{CB}} = \frac{\eta LWP}{4\rho_w [1 + (b/r_v)^4]} \quad (13)$$

We further approximate by assuming that the R^{-4} term is small compared to the first for the upper limit, i.e. $\frac{1}{4}(a/R_{CB})^4 \ll 1.1R_{CB}$ which is a good approximation for $R_{CB} \geq 30 \mu\text{m}$ which is the case for all cases with drizzle greater than a few hundredths of a mm day⁻¹. Then (12) yields an expression for the growth of the embryonic drizzle drops from cloud top to base:

$$\Delta R = R_{CB} - R_{emb} = \frac{\eta LWP}{4\rho_w [1 + (b/r_v)^4]} - \frac{a^5}{4R_{emb}^4} \quad (14)$$

Given the assumptions made regarding the cloud droplet size distribution, i.e. model assumption 2 (see Section 2a), a simple parameterization for R_{emb} as a function of r_v is possible, viz. $R_{emb} \approx c(r_v + d)$, with $c = 1.4$ and $d = 4.0 \mu\text{m}$ (note that the values of c and d will depend upon the assumed precipitation embryo concentration N_D , here assumed to be 100 liter⁻¹ and should be tuned if N_D is varied). Then (14) becomes

$$\Delta R = R_{CB} - R_{emb} \approx \frac{\eta LWP}{4\rho_w [1 + (b/r_v)^4]} - \frac{a^5}{4c^4(r_v + d)^4} \quad (15)$$

The precipitation rate at cloud base from the minimal model $\tilde{P}_{CB}$ is then given by

$$\tilde{P}_{CB} = \frac{4\pi\rho_w}{3} \alpha_T N_D R_{CB}^{3+\beta} \quad (16)$$

$$= \frac{4\pi\rho_w}{3} \alpha_T N_D \left\{ c(r_v + d) + \frac{\eta LWP}{4\rho_w [1 + (b/r_v)^4]} - \frac{a^5}{4c^4(r_v + d)^4} \right\}^{3+\beta} \quad (17)$$

Equation 17 expresses the cloud base precipitation rate as a function only of the cloud liquid water path and the mean volume radius at cloud top (uniquely specified as a function of the LWP and N_d).

The first term in the parentheses represents a memory of the initial size of the embryonic drizzle drops. The second term represents the reservoir of available cloud water to be collected by the falling drizzle drops (with the denominator describing limits to their growth when the collected droplets are small and therefore collected inefficiently), and the third term is an suppression that represents inhibition by the limited *initial* size of the falling embryos themselves.

Fig. 12 shows that the minimal model cloud base precipitation rate P_{CB} agrees very well with that from the full CC model and is able to demonstrate the increasing importance of LWP in determining the precipitation rate at low N_d and high LWP . Equation 17 readily demonstrates sensitivity to both LWP and N_d consistent with observations in stratocumulus clouds (Pawłowska and Brenguier 2003; Comstock et al. 2004; Van Zanten et al. 2005) but for deeper precipitating trade cumulus clouds (which typically have higher local LWP and low N_d), the sensitivity to LWP increases and that N_d decreases so that $\tilde{P}_{CB} \sim LWP^{3+\beta}$.

To further indicate the utility of the minimal model, Figure 13 shows the relative importance of the three terms in (17) superimposed on the precipitation rate from the CC model. As the precipitation rate increases, the term involving the accretion of cloud water becomes dominant. This also demonstrates that the cloud top effective radius alone does not serve as a particularly useful predictor of the tendency to precipitation or of the rate that a cloud of a given thickness might produce. Only for the very smallest P_{CB} and for very low LWP are the precipitation isohyets even close to being parallel to the lines of constant effective radius.

For these thicker clouds, where h is significantly larger than z_0 , LWP is approximately linearly dependent upon the cloud thickness. This is consistent with an observed lack of a trend in the liquid water profile with height for clouds thicker than about 1 km (Raubert et al. 2007). Thus the CC model suggests that the precipitation rate in trade cumuli would scale with the cloud thickness to the fourth or fifth power, with a much weaker dependence upon N_d , a result that is at least qualitatively consistent with recent large eddy simulations (Stevens and Seifert 2008).

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TABLE 1. Input and free-parameters for the two models in this study

Parameter type	Model Type	
	Continuous Collection (CC)	Steady State (SS)
Primary macrophysical	Cloud thickness h (or LWP)	Cloud thickness h (LWP is prognostic)
Primary microphysical	Cloud droplet concentration N_d	Cloud droplet concentration N_d
Others (default values in parentheses)	Adiabaticity scale height z_0 ($z_0 = 500$ m)	Turbulent replenishment timescale τ_{rep} ($\tau_{rep} = 1$ hour)
	Drizzle drop concentration N_D ($N_D = 100$ liter $^{-1}$)	Drizzle drop embryo mass m_{emb} ($m_{emb} = 4.46 \times 10^{-11}$ kg)

TABLE 2. Regions used for observational/model comparison in this study, together with parameters indicating the macrophysical and microphysical cloud properties determined using the A-Train data described fully in Kubar et al. (2009). Only clouds over open ocean are included. **Bold** numbers represent the maximum value among all regions, and *italics* represent minimum values. All numerical values other than percentages represent median values for the region.

Region	Latitude	Longitude	Cloud top height (km)	LWP (g m ⁻²)	r_e^+ (μ m)	N_{eff} (cm ⁻³)
NE Pacific	15-30°N	150°E-130°W	2.0	177	16.7	44
Far NE Pacific	15-30°N	105-125°W	1.4	138	13.9	71
Gulf of Mexico	20-30°N	70-100°W	2.3	250	12.2	127
Asian Coast	15-30°N	100-130°E	2.2	264	<i>11.5</i>	159
SE Pacific	15-30°S	90-150°W	2.1	161	18.0	<i>33</i>
Far SE Pacific	0-30°S	70-80°W	<i>1.3</i>	<i>108</i>	13.2	72
Deep convective	5-15°N	140°E-140°W	2.7	216	17.1	44
	5-15 °S	150°E-130°W				
Eq. cold tongue	5°S-5°N	85-130°W	1.7	155	14.6	61

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