

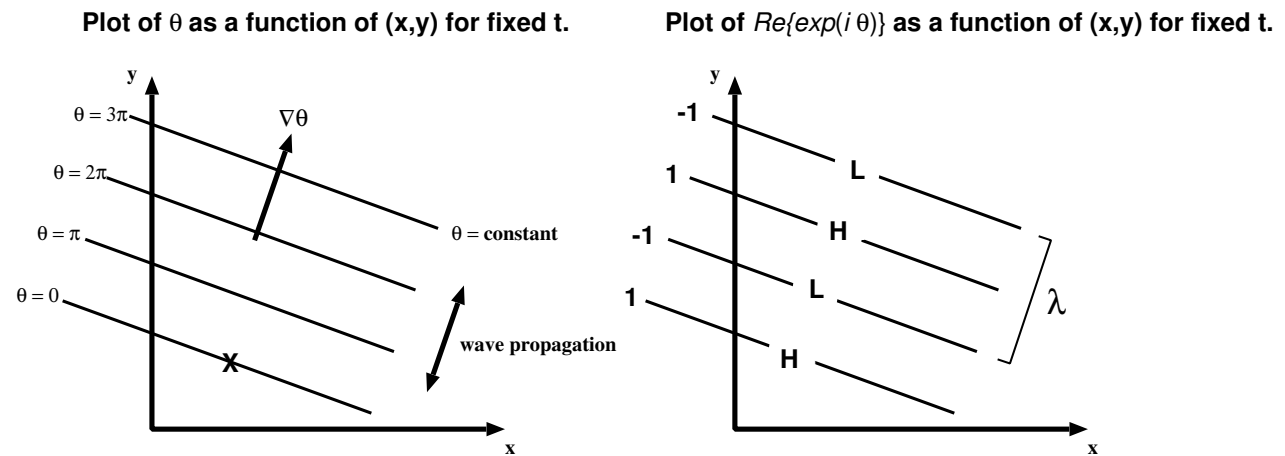
# Plane-Wave Summary

A two-dimensional plane wave may be expressed as

$$f(x, y, t) = \text{Re} \{ A e^{i(kx+ly-\nu t)} \} = \text{Re} \{ A e^{i\theta} \} \quad (1)$$

- $x, y$  and  $t$  are independent variables (space and time).
- $k$  and  $l$  are the  $x$  and  $y$  wavenumbers (units:  $\text{m}^{-1}$ ).
- $A$  is the wave *amplitude*.
- $\theta = kx + ly - \nu t$  is the wave *phase angle*.
- The wave *propagates* normal to lines of constant phase angle.

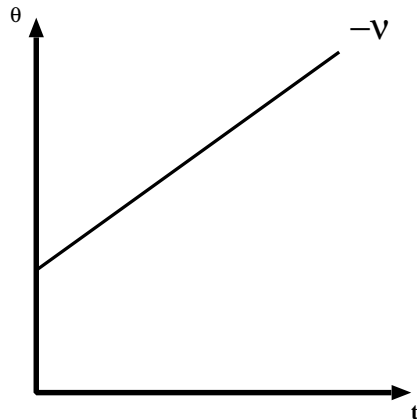
**At any instant in time** [ $t$  fixed;  $(x, y)$  varies]:



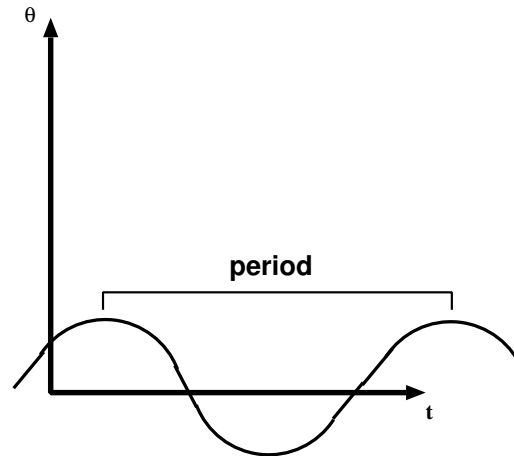
- $\theta = kx + ly + C$ ;  $\theta$  is a linear function of space.
- $\theta$  is constant on lines of  $kx + ly$ .
- $e^{i\theta} = e^{i(\theta+2\pi n)}$ , where  $n$  is an integer, are lines of *constant phase* (e.g. highs and lows).
- $\vec{K} = \nabla\theta = \hat{i}k + \hat{j}l$  is the *wave vector*;  $\mathcal{K} = |\vec{K}|$  is the *wavenumber*.
- $\lambda = \frac{2\pi}{\mathcal{K}}$  is the *wavelength*: the distance between lines of constant phase.

At any fixed point in space  $[(x, y) \text{ fixed}; t \text{ varies}]$ :

Plot of  $\theta$  as a function of  $t$  for fixed  $(x, y)$ .



Plot of  $\text{Re}\{\exp(i\theta)\}$  as a function of  $t$  for fixed  $(x, y)$ .



- $\theta = C - \nu t$ ;  $\theta$  is a linear function of time.
- $\nu = -\frac{\partial\theta}{\partial t}$ , is called the *frequency*: the rate that lines of constant phase pass a fixed point in space (units:  $\text{s}^{-1}$ ). Note that the figure above indicates  $\nu < 0$ . This means that for fixed  $(x, y)$ , such as the point marked “**X**” on the first figure,  $\theta$  increases with time; this can only occur if phase lines move toward smaller  $x$  and  $y$ .
- The wave *period* is  $\frac{2\pi}{\nu}$ : length of time between points of constant phase (units: s).
- The *phase speed* is the propagation speed of constant phase lines in the direction of  $\vec{K}$ ,  $c = \frac{\nu}{\mathcal{K}} = -\frac{1}{|\nabla\theta|} \frac{\partial\theta}{\partial t}$  (units:  $\text{m s}^{-1}$ ).

### Special note on $\theta$ :

If  $\theta$  has an *imaginary part*,  $\theta = \theta_r + i\theta_i$ , then  $e^{i\theta} = e^{i(\theta_r + i\theta_i)} = e^{i\theta_r} e^{-\theta_i} \equiv A^* e^{i\theta_r}$ .  $\theta_r$  is the wave phase angle as interpreted above, and  $A^* = Ae^{-\theta_i}$  is a modified amplitude that depends on time and/or space. For example, if the frequency,  $\nu$ , contributes the imaginary part, then the wave has *time-dependent* amplitude that grows or decays with time. Such waves are called *unstable*, to distinguish them from the *neutral* waves ( $A$  constant) that we discussed above.