

ATMS542 Summary Review

Here is a non-exhaustive list of some of the essential “take home” messages from ATMS542. The list is in chronological order, without regard to priority of importance.

(0) In the extratropical latitudes of Earth, we find persistent jet streams and disturbances (i.e. baroclinic Rossby waves and cyclones) on these jet streams that are responsible for weather and affect climate. A goal of this class was to develop a framework to explain the existence and dynamics of these features.

(1) The hydrostatic primitive equations (PE) support two types of modes: Rossby waves (RWs) and internal gravity waves (GWs). The RWs are slower than the GWs and they have potential vorticity (PV), which the gravity waves do not. Furthermore RWs have relatively small divergence and vertical motion compared to the GWs. These differences allow us to reduce the PE to a simpler set of equations that are completely determined by PV (the QG equations). The fact that the dynamics only depend on a single time derivative (as compared to three for the PE) means that we can solve problems and gain physical insight much more easily than we can for the PE. These equations provide the foundation for our understanding of extratropical dynamics.

(2) The QG equations support two frameworks for understanding dynamics: one based on vertical motion (“ w thinking”), and another based on PV (“PV thinking”). w thinking involves the vorticity and thermodynamic equations, and changes to these quantities due to w , which is directly linked to frontogenesis; w may be calculated from Q -vectors or estimated by the advection of vorticity by the thermal wind. PV thinking involves only the inversion and advective re-distribution of PV. Inversion is the process whereby all other quantities, such as wind, are recovered from the PV. With a suitably chosen “mean state,” PV anomalies may be defined, and one may quantify “interaction” dynamics between these features, and with the mean state; e.g., RWs and vortices.

(3) Disturbances may spontaneously grow on QG shear flows. The two primary mechanisms for this growth are baroclinic and barotropic instability. The main distinction between these instabilities is drawn from the QG energy equation, which states that disturbances grow baroclinically (barotropically) when they tilt against a mean-state vertical (horizontal) shear. In addition to this exchange of energy with the mean state, disturbance energy

may be transported (“fluxed”) to other locations; this process is important for downstream development. Individual eddies may originate with either baroclinic conversion or eddy flux convergence; the former (latter) usually occurs in the upstream (downstream) end of storm tracks. Eddies decay primarily through eddy flux divergence, but also through horizontal momentum fluxes that accelerate the mean flow. In the volume integral, the eddy flux integrates away, and we are left with a “four box” picture for the flow of energy in the atmosphere: potential energy is restored to the mean state by solar insolation and longwave radiation to space; eddy potential energy is created by poleward heat flux; eddy kinetic energy is created by upward heat flux; mean-state kinetic energy is restored by horizontal momentum fluxes; mean-state and eddy kinetic energy are lost to friction.

Eddy interactions with the mean flow were quantified in the zonal mean using the Eulerian mean (EM) and transformed Eulerian mean (TEM) equations. From the EM perspective, the zonal-mean mean flow changes due to eddy fluxes of heat and momentum, and from mean-meridional circulations. Because the latter are driven by the former, and there is a cancellation between the explicit and implicit eddy forcing, the TEM equations replace the mean-meridional circulation with a “residual” mean-meridional circulation. This approach shows that cancellation can be complete, resulting a mean-flow unaffected by the eddy fluxes, which was quantified in a non-acceleration theorem.

(4) Observations show that the troposphere has nearly uniform PV, and is approximated well by a simple model having zero PV gradients and a rigid tropopause (the Eady model). An analysis of the linear stability problem for this model shows that plane waves exhibit exponential growth provided that their wavelength is sufficiently long with respect to the depth of the troposphere; the wavelength of maximum growth for typical midlatitude parameters is about 4000 km. These normal modes have no PV, time independent structure, and move eastward at the speed of the midlevel mean flow. At wavelengths shorter than the shortwave cut-off, the normal modes are neutral, with one based at the tropopause, and the other based at the surface. The growth/decay and propagation of these modes may be understood in terms of both w thinking and PV thinking. A separate set of solutions, singular neutral modes, move at the speed of the wind at a given level in the troposphere, and have a spike of PV at this level. Together, the normal and singular neutral mode solutions span the solution space for the Eady model; that is, for *any* initial condition, the solution for any later time is given by a linear combination of these modes.

The normal and singular neutral mode solutions look like $\phi(x, z, t) = \hat{\phi}(z)e^{i(kx - \omega t)}$; notice that the spatial structure is independent of time. If we allow for spatial structure that depends on time, we may recast the stability problem over a finite time interval, and look for solutions that grow *optimally* for the chosen interval as measured by a chosen norm.

These solutions, called singular vectors or optimal modes, often amplify much more than the most unstable normal mode over the same time interval. Singular vectors also span the solution space for the Eady model, so they must be related to the normal and singular neutral mode solutions. For example, we may interpret the optimal modes as a linear combination of appropriately weighted normal and singular neutral modes. This view links to a simple illustration of the essential aspects of optimal-mode amplification: a sum of two neutral modes (either normal or singular) that are not orthogonal in the chosen norm. Although this analysis generalizes our original results on growth, it is sensitive to the chosen time interval and norm, which are arbitrary.

(5) Barotropic instability involves the growth of disturbances due to horizontal shears in the mean state. The problem geometry, solutions, and interpretations are very similar to the baroclinic instability problem. This similarity is strongest when viewed from the PV thinking framework, in terms of RWs propagating on PV gradients and mutually amplifying in the case of the growing unstable modes. An important difference involves the growth rate, which in both cases depends on the shear. For baroclinic instability, the shear over the depth of the troposphere is limited by the strength of the jet stream; i.e., the pole–equator temperature gradient. For barotropic instability, the horizontal shear may be much larger, because the width of the shear zone is essentially unlimited. For example, if we have opposing currents of wind separated by distance L , then the shear increases like L^{-1} . In addition to the observational examples given in class, you should be aware that barotropic instability may play an important role in large scale, low-frequency variability due to the full (complicated) spatial variability of the jet stream; the details of this process are beyond the scope of this course.

(6) Nonlinear processes become important whenever disturbances are not small in some appropriate measure. Nonlinearity is associated with the sensitivity to initial conditions (“chaos”) and inertia dynamics (“turbulence”). For the barotropic vorticity equation on a β plane, we found a parameter, $\delta = \frac{U}{\beta L^2}$, that scaled the importance of nonlinear advection. This parameter tells us that nonlinearity increases for small scales ($\sim L^{-2}$) and large amplitudes ($\sim U$). There exist exact, steady, solutions for all values of δ . For $\delta \ll 1$, there are linear (Rossby) waves. For $\delta \gg 1$, there are nonlinear vortices and general solutions of the form $\zeta = f(\psi)$. For $\delta \sim 1$ there are solutions in the form of weakly nonlinear solitary waves that we have not talked about. In the general, unsteady, case, we typically have a mixture of waves and turbulence. For barotropic dynamics, this implies an upscale cascade of energy to a scale where the dynamics are dominated by linear processes.