Estimating the Response of Extreme Precipitation over Mid-latitude Mountains to Global Warming

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Global warming induced changes in extreme orographic precipitation are investigated using a hierarchy of models: a global climate model, a limited-area weather forecast model, and a linear mountain-wave model. We consider precipitation changes over an idealized north-south mid-latitude mountain barrier at the western margin of an otherwise flat continent. The intensities of the extreme events on the western slopes increase by \( \sim 4\% K^{-1} \) of surface warming, close to the “thermodynamic” sensitivity of vertically integrated condensation in those events due to temperature variations when vertical motions stay constant. In contrast, the intensities of extreme events on the eastern mountain slopes increase at \( \sim 6\% K^{-1} \). This higher sensitivity is due to enhanced ascent during the eastern-slope events, which can be explained in terms of linear mountain-wave theory as arising from global-warming induced changes in the upper-tropospheric static stability and the tropopause level. Similar changes to these two parameters also occur for the western-slope events, but the cross-mountain flow is much stronger in those events, and as a consequence, linear theory predicts no increase in the western-slope vertical velocities. Extreme western-slope events tend to occur in winter, whereas those on the eastern side are most common in summer. Doubling \( \text{CO}_2 \) not only increases the precipitation, but during extreme western slope events it shifts much of the precipitation from snow to rain, potentially increasing the risk of heavy runoff and flooding.
1. Introduction

In contrast to the global-warming-induced changes in mean precipitation predicted by climate models, which decrease in certain regions such as the subtropics, precipitation extremes are expected to increase in almost all areas of the globe (Emori and Brown 2005; Kharin et al. 2007). Extratropical precipitation extremes in climate simulations increase at about $6 \% K^{-1}$ of globally averaged surface warming. This rate of increase is close to the ‘thermodynamic’ sensitivity of condensation to warming, which is produced by temperature increases at fixed relative humidity when vertical motions stay almost constant (O’Gorman and Schneider 2009a). The simulated sensitivity of tropical precipitation extremes differs substantially between climate models. Nevertheless, by applying observational constraints to climate simulations and exploiting the relationship between the simulated responses to interannual variability and climate change, O’Gorman (2012) estimated a sensitivity of the 99.9th percentile of daily tropical precipitation to climate change at $10 \% K^{-1}$.

With 26% of the world’s population living within mountains or their foothills, and over 40% living in river basins originating in mountainous regions (Beniston 2005), understanding the response of orographic precipitation extremes to global warming is important for anticipating societal impacts such as flooding and landslides (Maddox et al. 1978; Lin et al. 2001; Rasmussen and Houze 2012). Yet only a few previous studies have reported on warming-induced precipitation extremes over mountains as part of more widely targeted numerical simulations. These studies (Diffenbaugh et al. 2005; Singh et al. 2013; Wehner 2013) suggest that precipitation extremes will increase in frequency and intensity over the high elevations and rain-shadowed regions of the Pacific Northwest portion of the United States.

Here we use a combination of global-climate and mesoscale-weather-forecast models, together with a linear mountain wave model, to develop a more complete physical understanding of the
processes governing changes in extreme precipitation over simplified topography representative of one of the north-south mountain ranges along the west coast of North America. We use the mesoscale model to provide more detailed simulations of the response of mid-latitude orographic precipitation extremes to global warming in the different environmental parameter regimes produced by the global climate model. The changes on both the the windward (western) side of the mountains, and on their leeward (eastern) side are partitioned into the contributions from thermodynamics and from dynamics in the spirit of O’Gorman and Schneider (2009a,b). The linear mountain wave model is used to explore the physical principles governing the dynamical response.

The paper is organized as follows. Section 2 describes the numerical models used in our idealized experiments. Section 3 documents responses of the climatological means of orographic precipitation to doubled CO$_2$. Section 4 examines the distribution and synoptic-scale structure of the extreme events. The relative contributions of thermodynamics and dynamics to the changes in extreme-event intensity are explored in Section 5. Section 6 examines the elevational dependence of the changes in extreme precipitation. Our conclusions are presented in Section 7.

2. Models and Methods

The numerical models used in our study are the Geophysical Fluid Dynamics Laboratory (GFDL) global HIgh Resolution Atmospheric Model (HiRAM) (Zhao et al. 2009) and the Weather Research and Forecasting (WRF) model Version 3.5.1 (Skamarock et al. 2008). The HiRAM model is run at approximately 50 km horizontal resolution to effectively create a multidecadal data set for both a control and a globally warmed climate. The most extreme orographic precipitation events from this larger sample are dynamically downscaled using the WRF model at 12-km horizontal resolution.
To isolate the dynamic and thermodynamic processes governing the changes in extreme orographic precipitation, we consider an idealization of the mountainous terrain in north America, along with the drier plains to their east. Four copies of these prototypical north-American continents are distributed at 90° intervals around the northern mid-latitudes (Fig. 1a). The remainder of the planet is covered with a 24-m deep mixed layer ocean. The continents extend 30° east-west in longitude and span the latitude band 30°–60°N. They are flat except on their west coasts, where a single smooth mountain ridge runs north-south just inland from the coast. The surface elevation $z_s$ of each ridge is determined by

$$z_s(x, y) = \frac{h_m}{2} \begin{cases} 1 + \cos(\pi r), & \text{if } |r| < 1, \\ 0, & \text{otherwise}, \end{cases}$$

where $h_m = 2.5$ km is the height of the ridge\(^1\), and $r(x, y)$ is defined as:

$$r(x, y) = \begin{cases} \sqrt{\frac{(x-x_0)^2}{a^2}}, & \text{if } |y-y_0| < b-a, \\ \sqrt{\frac{(x-x_0)^2 + [|y-y_0|-(b-a)]^2}{a^2}}, & \text{otherwise}; \end{cases}$$

where $a$ is the east-west half width of the ridge (set to 240 km in the HiRAM and 120 km in the WRF simulations); $b$ is the north-south half width (taken as 1675 km, corresponding to 15° in latitude, in all simulations) and $x_0$ and $y_0$ are the coordinates of the center of the ridge.

HiRAM has a finite-volume dynamical core using a cubed-sphere grid topology and sophisticated physics modules for simulating processes such as in cloud microphysics, moist convection and land surface exchanges. The same model parameters and resolution ($\approx 50$ km horizontally, 32 vertical levels) as those specified in Zhao et al. (2009) are adopted in our simulations. We run HiRAM with modern (330 ppm) and doubled (660 ppm) CO$_2$ concentrations and daily averaged solar insolation for 20 years, and retain 6-hourly data from the last 10 years for analysis. Since four

\(^1\)This is roughly the average height of Rocky Mountains, see Fig. 8a.
identical continents are symmetrically distributed around a latitude circle, we treat the 10 years of
global analysis data as a 40-year data set over a single continent. Our focus is precipitation on each
side of the ridge on the western edge of this continent. The sensitivity of precipitation to global
warming, for both time means and extremes, is defined as the percentage change in precipitation
rate divided by the global-mean surface temperature increase, which is 5 K is our simulations.

To select a group of extreme events over the mountains from each simulation, we need first
identify individual precipitation events and define their intensities. Different events can be sepa-
rated based on connectivity in latitude-time plots of the zonally averaged precipitation rate over
each east-west segment of grid cells ascending the western or descending the eastern slopes. The
latitude-time plots are smoothed by applying a 4-point (24 h) running average in time and a 3-point
(150 km) running average in latitude before measuring connectivity. Each event identified in this
way corresponds to an individual weather system passing over the mountains, and its intensity is
defined as the mean precipitation rate in the 24-hours and the 150-km north-south wide latitude
band on the eastern or western slope receiving the most precipitation during that event. Finally,
all precipitation events over the eastern and over the western mountain slopes are ranked based on
their intensities, and for each side of the ridge, the top 40 events in the control and $2 \times \text{CO}_2$
simulations are selected for further analysis. Since a significant fraction of the air is diverted laterally
around northern and southern ends of mountains, leading to different dynamical regimes in those
regions, our analysis is restricted the segment of mountains between 32.5° and 57.5°N.

To attain a more robust evaluation of the changes in extreme events, the top 10 events in each
40-event set of HiRAM extremes are re-simulated with WRF using higher resolution and narrower
mountains whose slopes better approximate average slopes in the real world. The WRF simula-
tions also use 18 more vertical levels than in HiRAM. In the WRF simulations, a 12-km resolution
domain is one-way nested in a 36-km domain, whose initial and lateral boundary conditions are
provided by the HiRAM data (Fig. 1b). The diurnal cycle is included in the WRF simulations, with the insolation determined by the calendar date of each individual event. The HiRAM fields are computed using daily averaged solar insolation; since the boundary of the outer WRF domain is entirely over the ocean, this did not lead to any incompatibilities. A list of the WRF physics schemes used in our simulations is provided in the Appendix. The WRF simulations run for 5-6 days centered around the time of the HiRAM event. The time, location, and intensity of the most intense 24-hour period in the fine-scale WRF run is determined in a manner similar to that for the parent HiRAM event except that the event precipitation is meridionally averaged over a 108-km (9-cell) latitude band.

Because of its higher resolution and superior microphysical parameterization, the WRF simulations provide better dynamical details of the extreme events than do HiRAM results. Nevertheless, the response of the extreme events to warming are quantitatively consistent in all major aspects in the WRF and HiRAM simulations. Unless otherwise stated, we use data from WRF simulations for the discussion about extreme events in sections 4 to 6.

3. Responses of climatological means

Before examining the extremes, we briefly consider annual mean and seasonal mean orographic precipitation, and its response to increased CO$_2$ in the full HiRAM dataset. For both the eastern and western slopes (halves) of the mountains, the annual and the seasonal mean precipitation are greatest over the center section of the ridge, and decrease to the north and south. Over the western slopes, the mean wintertime precipitation (December, January, February) is roughly twice that in summer (June, July, August), whereas the opposite is true over the eastern slopes.

Figure 2 shows the sensitivity to doubled CO$_2$ of the annual-, summer-, and winter-mean precipitation over the western and eastern slopes of the mountains. The annual- and seasonal-mean
western-slope precipitation sensitivities increase near the northern end of the mountains, decrease near the southern end, and vary smoothly with latitude in between (Fig. 2a). The wintertime western-slope sensitivities are more positive than those for the summertime and the annual mean, except near the southern end of the range. Shi and Durran (2014) conducted similar GCM simulations with annual mean insolation and a shallower slab ocean, and they found the sensitivity of western-slope orographic precipitation varies from $-9$ to $13\% K^{-1}$ between $35$ to $60^\circ N$, which is very close to the sensitivity of wintertime western-slope precipitation in our current simulations. Their analysis showed that the changes in the western-slope precipitation are produced by an almost spatially uniform thermodynamic increase due to the change in the lapse rate of saturation specific humidity and by a north-south asymmetry in dynamical changes produced by the poleward shift of storm tracks.

The sensitivity of the eastern-slope precipitation differs from that for the western-slope in that there is increasing summertime precipitation over the southern part of the mountain despite the poleward shift of storm track. Since significant precipitation is likely associated with some sort of eddy, one might expect the eastern-slope precipitation to respond to warming in a manner similar to the summertime pattern over the western slope, because both of them are affected by the same storm track shift.

To understand why the sensitivity of the eastern-slope precipitation in summer does not have the expected north-south asymmetry, we composite the 850 hPa level wind fields for significant summer precipitation events over the eastern slope within the red square at $35^\circ N$ shown in Fig. 3. Here ‘significant events’ are taken as those times when 6-hour precipitation rates averaged within the square are greater than $0.2 \text{mm} \text{h}^{-1}$. About 9% of the total summertime period is taken up by these events, and they produce nearly 3/4 of the total summertime rainfall.
Figure 3 compares the composite flow pattern for significant eastern-slope summertime events in the control and doubled CO$_2$ simulations. Unlike wintertime, when significant eastern-slope precipitation occurs downstream of the axis of an open trough, significant summertime precipitation events occur when a small low to the south produces easterly upslope flow into the target area. This easterly upslope flow at 35°N is weaker in the doubled CO$_2$ composite. As a consequence, the thermodynamic tendency toward increased precipitation is almost cancelled by weaker dynamical forcing, and the net sensitivity per significant event only increases by 0.8%/K. The actual increase in significant event precipitation is primarily due to a 3.3%/K increase in the frequency of these events, which results from the weakening of climatological mean westerlies as the storm track shift poleward. The remainder of the weaker summertime events behave in a similar way, with the sensitivities produced by changes in frequency and intensity for all events adding up to give a total sensitivity of about 6%/K at 35°N. In contrast the frequency of significant eastern-slope wintertime events is reduced by -5.8%/K, consistent with a northward shift in storminess. This reduction is a major factor, but not the only contributor to the net climatological decrease of about -3%/K in the wintertime precipitation over the eastern slope in the south.

4. Distribution and synoptic structure of the extreme events

We now return the focus to extreme events, beginning with the four sets of top 40 events (western side, eastern side, control and doubled CO$_2$) from the HiRAM simulations.

a. Location and frequency

The north-south distribution of 40 most extreme (99.75 percentile) west-side and east-side events from the HiRAM simulation are shown in Fig. 4, in which the latitude of the row of grid cells along the mountain slope receiving the maximum 24-hour precipitation is binned in 2.5°-wide latitude
bands along the length of the topography. Extreme events occur most frequently over the central part of the mountain range.

The mean latitude at which the western-slope extremes occur shifts northward by 0.9° in the doubled CO$_2$ simulation, while the mean latitude for the eastern-slope extremes shifts southward by 1.1°. The northward shift in the western slope events might be linked to the northward shift in the storm track in the warmer world (Yin 2005). Nevertheless, with only 40 events in our sample, the north-south distribution of these events is noisy. Applying the Kolmogorov-Smirnov test (Miller 1956; Marsaglia et al. 2003) to a null hypothesis that the latitudinal distribution of extremes shown in Fig. 4 does not change in the warmer climate, we found the hypothesis of no change could not be rejected at the 5% significance level.

The precipitation thresholds that distinguish the 10 and the 40 most extreme events in the control climate are exceeded far more frequently in the doubled CO$_2$ simulation. Table 1 shows that on both the western and eastern slopes, the number of events that exceed the control-climate 40-event threshold increases by about a factor of 3 in the doubled CO$_2$ simulations. For just those events above the top-10 threshold, the increase in the warmer climate is more than a factor of 4.

b. Synoptic-scale structure

Western-slope and eastern-slope precipitation extremes tend to occur at different times in a year. As shown in Fig. 5, western-slope extremes mostly occur in winter months, whereas eastern-slope extremes are more likely to occur in the warm season. Fig. 6 shows snapshots of Water Vapor Path (WVP, the column integrated water vapor) and geopotential height at 500 hPa in a western-slope and an eastern-slope event. Though they are from individual cases, the synoptic patterns in Fig. 6 are very typical of the most extreme events in both the control and doubled-CO$_2$ climates. Unless
specified otherwise, the data for this and the subsequent discussions are taken from the sets of
top-ten events simulated with the WRF model on the 12-km fine mesh.

The western-slope extreme event is produced by a so-called “atmospheric river,” a narrow fil-
ament of concentrated moisture carried poleward from the sub-tropics (Zhu and Newell 1994).
Recent studies based on observational data from Western Europe and along the West Coast of the
United States, show atmospheric rivers are often responsible for heavy and extreme precipitation
(Ralph et al. 2006; Warner et al. 2012; Lavers et al. 2012; Lavers and Villarini 2013). As ap-
parent in Fig. 6a, the axis of the atmospheric river is nearly perpendicular to the mountain, and
it coincides with a jet of strong cross-mountain flow as evident from the strong gradient in the
geopotential height field. Strong orographic lifting condenses large amounts of water vapor that
precipitates out over the mountain. The contrast between the atmospheric river’s WVP to east and
west of the mountain clearly shows this drying.

The eastern-slope extreme event in Fig. 6b is caused by a cut-off low to the southwest of the
precipitation site, which produces strong southeasterlies over the eastern slope of the mountain.
A plume of concentrated moisture is embedded in the southeasterly flow. A very high-amplitude
ridge keeps the main jet stream well to the north of the cut-off low. Fig. 7 shows a sounding from
the location of the blue dot in Fig. 6b, upstream of the eastern slope. The atmosphere between 850
and 500 hPa is completely saturated and nearly moist neutral; the upslope easterly winds in this
layer are roughly 18 m s\(^{-1}\) (35 kn).

While flow patterns like that shown in Fig. 6b are less common in the real world than western-
slope atmospheric river events, close atmospheric analogs do occasionally occur. For example,
the record-breaking rainfall across the Colorado Front Range between September 11th and 13th in
2013 (Schwartz 2014) occurred in a synoptic setting resembling that in Fig. 6b. The basic structure
of the geopotential height field for the Colorado event, shown in Fig. 8a is similar to that in Fig. 6b,
with a cut-off low south of a high-amplitude ridge. The 1200 UTC sounding from Denver, CO on 12 September 2013 (Fig. 8b) also shows a similar deep layer of saturated, neutrally stratified flow, although the easterly winds in that layer are somewhat weaker than those in Fig. 7.

5. Changes in extreme-event intensity

Perhaps surprisingly, the mean sensitivity of the precipitation in the top 10 events to global warming is about $2\%\text{K}^{-1}$ higher on the eastern side (where it is $5.9\%\text{K}^{-1}$) than on the west side (where it is $4.2\%\text{K}^{-1}$). These values are comparable to the sensitivities for the vertically integrated condensation rate in the air columns at the same time and location of each of the extreme events, which are $6.5\%\text{K}^{-1}$ and $4.7\%\text{K}^{-1}$ for the eastern and western sides, respectively.

The differences between the sensitivities in precipitation and condensation are due to changes in the precipitation efficiency, defined as the event-integrated precipitation divided by the event-integrated condensation. The precipitation efficiencies in these extreme events over our 120-km half-width mountains are relatively high, ranging from an average of 0.82 for the 10-western slope events in the doubled-CO$_2$ climate to 0.96 over the eastern slope in the control climate. These precipitation efficiencies are likely high because of the broad width of our mountains and to the absence of embedded convection (Cannon et al. 2012). On both sides of the mountain, the precipitation efficiency decreases modestly as the planet warms; the sensitivities of precipitation efficiency being $-0.7\%\text{K}^{-1}$ and $-0.4\%\text{K}^{-1}$ over the eastern and western sides, respectively. These sensitivities are qualitatively consistent with the findings in Kirshbaum and Smith (2008) that precipitation efficiency decreases with increases in the local temperature upstream of idealized horizontally-uniform moist flow over a three-dimensional ridge.

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$^2$Top 40 events from the HiRAM simulations show similar sensitivities: $6.3\%\text{K}^{-1}$ over the eastern slopes and $3.9\%\text{K}^{-1}$ in the west.
a. Partitioning the sensitivities into thermodynamic and dynamic contributions

In our simulations, the sensitivity of the precipitation to changes in global mean temperature are dominated by the sensitivity of the event-integrated condensation. We therefore focus further analysis on those factors responsible for the changes in condensation, separating the effects of thermodynamic and dynamical changes in the condensation during those events. The local condensation rate $c$ in the extreme precipitation events may be estimated by assuming moist adiabatic lifting maintains the water vapor content of the rising air at saturation, i.e.

$$c = -w \frac{dq_s}{dz} = w \gamma_s,$$

where $w$ is vertical velocity, $q_s$ is saturation specific humidity, and $\gamma_s = -dq_s/dz$ is the lapse rate of saturation specific humidity\(^3\). Over 95% of the precipitation in the extreme orographic precipitation events in the 12-km WRF simulations is produced by grid-resolved physical processes; the remainder (less than 5%) is from parameterized convection. Thus (3) can be applied directly to the archived model data to approximate the condensation rate in each saturated grid cell. Integrating (3) vertically, one gets the total condensation in an air column,

$$C = \sum_k s_k c_k,$$

where the summation is limited to saturated grid cells by defining

$$s_k = \begin{cases} 
0 & \text{if unsaturated,} \\
\Delta p_k / g & \text{otherwise,} 
\end{cases}$$

here $\Delta p_k$ is the pressure thickness of layer $k$ and $g$ is the gravitational constant. One can further average $C$ over time and area for each extreme event (the 24 hours and the $108 \times 108$-km square on the mountain slope receiving most precipitation—denoted by a overbar), and then average results

\(^3\)An analytic expression for $\gamma_s$ as a function of temperature and pressure is provided in Shi and Durran (2014)
of the 10 events in either the control or the doubled-CO$_2$ climate (denoted by angle brackets) to obtain the mean condensation rate for the extremes

$$\langle \bar{C} \rangle = \langle \sum_k s_k c_k \rangle. \tag{6}$$

Letting subscripts ‘w’ and ‘c’ denote the doubled-CO$_2$ and control climates, the change in mean condensation due to warming is

$$\delta \langle \bar{C} \rangle = \langle \bar{C} \rangle_w - \langle \bar{C} \rangle_c \tag{7}$$

If one could pair the precipitating columns in a control climate event with the columns in a warmed climate event so that each pair of columns have the same saturated levels and the same time and space distribution, one could separate $\delta \langle \bar{C} \rangle$ into two contributions

$$\langle \sum_k s_k [ (c_k)_w - (c_k)_c ] \rangle = \langle \sum_k s_k \delta (w \gamma_s)_k \rangle$$

$$\approx \sum_k s_k \left[ (w \delta \gamma_s)_k + (\gamma_s \delta w)_k \right], \tag{8}$$

where $\delta$ again denotes the change between warm- and control-climate values. In (8), the terms involving $w \delta \gamma_s$ and $\gamma_s \delta w$ would estimate the contributions from thermodynamical and dynamical changes, respectively. Such pairing is, however, not possible because the number of saturated layers is typically greater in the warmer climate.

Nevertheless, we can still estimate the two terms on the right side of (8) as follows. The thermodynamic contribution is evaluated as

$$\delta \langle \bar{C} \rangle_{\text{thrm}} = \frac{\sum_k (s_k w_k)_w \delta \langle \gamma_s \rangle_k + \sum_k (s_k w_k)_c \delta \langle \gamma_s \rangle_k}{2}. \tag{9}$$

During these extreme events, the lapse rate of saturation specific humidity varies with height, but is almost uniform in the horizontal over each 108×108-km square and 24 hour period, so its change is well captured by $\delta \langle \gamma_s \rangle_k$. The same is not true for the vertical velocity, which varies systematically in the cross mountain direction and also exhibits substantial fluctuations in the along-slope
direction. Within each 108 km north-south strip, the extreme precipitation is dominated by the
cells with the strongest vertical velocities. Therefore, in estimating the dynamic contribution to
(8), the simple time and space average used to obtain $\bar{\gamma}_k$ is replaced by $\bar{\tilde{w}}_i$, where $i$
indexes the cells in the cross-ridge direction, and for any given $i$, $\bar{\tilde{w}}_i$ denotes the time average of the
maximum $w$ in the along-ridge direction. The dynamic contribution is then computed as

$$
\delta \langle \mathcal{C} \rangle_{\text{dyn}} = \frac{\sum_{i,k} (s_{i,k} \gamma_{s,i,k}) \tilde{w}_i}{2N_i},
$$

where $N_i = 9$ is the number of cells in the east-west direction included in the event average, and
the $y,t$ notation at the end of the overbar denotes event averages taken in the 108 km north-south
direction and over the 24-hour time period.

Over the eastern slopes, $\delta \langle \mathcal{C} \rangle_{\text{thrm}}/\langle \mathcal{C} \rangle_c = 3.9 \% K^{-1}$, $\delta \langle \mathcal{C} \rangle_{\text{dyn}}/\langle \mathcal{C} \rangle_c = 3.2 \% K^{-1}$, and their sum
provides a good approximation to the true value of $\delta \langle \mathcal{C} \rangle/\langle \mathcal{C} \rangle_c = 6.5 \% K^{-1}$ (exceeding it by 9%).

Over the western slope the same thermodynamic and dynamic sensitivities are 5.0 \% $K^{-1}$ and
0.03 \% $K^{-1}$, whose sum differs from the total sensitivity 4.7 \% $K^{-1}$ by just 6%. The thermody-
namic sensitivity is somewhat stronger over the western slopes of the mountains than over the
eastern slopes, which might be expected because most western-slope extreme events occur during
winter while the eastern-slope extremes are mostly summertime phenomena, and $\partial (\ln \gamma_s)/\partial T$ is
larger at colder temperatures (Shi and Durran 2014; Siler and Roe 2014).

By far the largest east-west contrast in sensitivities is in the dynamical contribution. The large
eastern-slope dynamic sensitivity more than compensates for its weaker thermodynamic sensitivity
and is responsible for the eastern side having a much larger value of $\delta \langle \mathcal{C} \rangle/\langle \mathcal{C} \rangle_c$ than the western
side. A closer look at this dynamical enhancement is provided in the next section.
b. Source of the dynamical enhancement

The dynamical contribution to the changes in extreme events arises from the differences between \( \tilde{w}_i \) in the control and warmer climates. These differences are illustrated by the vertical profiles of \( \tilde{w}_i \) in Fig. 9 for three different cross-ridge locations above the eastern and the western slopes. Consistent with the negligible value of \( \delta \langle C \rangle_{\text{dyn}} / \langle C \rangle \) over the western slopes, Fig. 9a-c shows almost no change in the vertical velocity profiles between the control and the warmer climates. In contrast, on the eastern side, \( \tilde{w}_i \) increases significantly as the climate warms at the lower and mid-slope elevations (Fig. 9d,e).

The vertical motions in extreme precipitation events over mountains are governed by mountain wave dynamics, as illustrated by the vertical-velocity cross-sections passing through the updraft cores for four representative events plotted in Fig. 10. In these moist events, the effective stability in the middle and lower troposphere is weak, so the vertical wavelength of the waves is long and the axes of their updrafts and downdrafts extend almost straight upward to a height around 7 km. The upstream tilt with height of the phase lines in these waves is more pronounced in the stratosphere and upper troposphere. As indicated by the green contours, most of the condensation responsible for the precipitation extremes occurs at low levels in the updrafts above windward slopes.

The depth and intensity of this windward ascent depends on the propagation and reflection of mountain waves. In particular, Siler and Durran (2015), using linear theory and numerical simulations, showed that the partial reflection of mountain waves at the tropopause can have an important impact on the vertical motions over mountains and the resulting precipitation. Here we compare the lower-tropospheric vertical velocities forced by the mean background atmospheric structures during the extreme events in the control and warmer climates. We compute these velocities using the linear hydrostatic mountain-wave model of Klemp and Lilly (1975), which assumes the atmo-
sphere is incompressible and consists of three layers with constant static stabilities $N$ and constant
wind shear in each layer.

For the eastern-slope means, we neglect the vertical variations of the winds with height, which is
a reasonable approximation to the actual average upstream flow. Those relatively uniform easterly
winds extend up through troposphere until finally encountering a critical level (where the cross-
mountain easterlies drop to zero) in the lower stratosphere.\(^4\) In the context of the linear model,
wave absorption at the critical level is represented by an upper boundary condition requiring all
wave energy propagation be upward.

Assuming a vertically uniform mean cross-mountain wind $U$, the steady-state two-dimensional
solution in the lowest layer for a single Fourier component $w_L = \Re\{\hat{w}_1(z) e^{ikx}\}$ is

$$\hat{w}_1 = ikUh_m \left[ \cos \left( \frac{N_1}{U} z \right) + \alpha \sin \left( \frac{N_1}{U} z \right) \right], \tag{11}$$

where

$$\alpha = \frac{N_1 \sin \phi_1 + N_2 \beta \cos \phi_1}{N_1 \cos \phi_1 - N_2 \beta \sin \phi_1}, \quad \beta = \frac{N_2 \sin \phi_2 + iN_3 \cos \phi_2}{N_2 \cos \phi_2 - iN_3 \sin \phi_2},$$

and

$$\phi_j = \frac{N_j}{U} (z_j - z_{j-1}) \quad (j = 1, 2).$$

The subscripts in the above expressions denote the respective layers; $h_m$ is the mountain height
(2.5 km); $z_j$ is the elevation of the top of layer $j$, and $z_0 = 0$. We shall consider only the solution
for a single wave number $k$ that matches the mountain width ($2a = 240$ km) in our WRF simula-
tions, though the solution for a particular mountain shape can be readily constructed using Fourier
transforms.

During extreme precipitation events the atmosphere has a three-layer structure formed by a
saturated and moist nearly neutral lower troposphere, topped by the dry upper troposphere and

\(^4\)As an example, see the winds for one specific event in Fig. 7.
the stratosphere, with the stratosphere having the largest stability. The air in the lowest layer may become unsaturated on the lee side of mountains due to descent, but previous studies suggest that despite the presence of such unsaturated regions, the effective bulk static stability of the lowest layer \( N_1 \) is very close to the moist Brunt-Väisälä frequency (Jiang 2003; Siler and Durran 2015).

In addition, we found that for our specific application, there was little sensitivity to doubling or halving \( N_1 \).

Following Durran (1992), who suggested the linear tropopause tuning criteria of Klemp and Lilly (1975) works best when adjusted for finite-amplitude mountains by setting \( z_2 \) equal to the actual tropopause height minus \( 3h_m/2 \), in the control and warmer climates, \( z_2 \) is specified as 8.0 and 8.5 km, respectively. The mean upper tropospheric moist static stability gives \( N^2_2 \) as \( 0.94 \times 10^{-4} \) and \( 1.15 \times 10^{-4} \, \text{s}^{-2} \) in the control and warmer climates. Increases in mid-latitude static stability are a robust response to global warming in climate simulations (Frierson 2006), and the rise in tropopause height due to CO\(_2\) increases has been seen in both observations and climate model simulations (Kushner et al. 2001; Santer et al. 2003). The depth of the saturated layer is well approximated as \( z_1 = z_2 - 5 \) km. The other parameters required to evaluate (11) remain almost the same in the control and warmer climates; we estimate these values as \( N^2_1 = 2.5 \times 10^{-5} \, \text{s}^{-2} \), \( N^2_3 = 4.0 \times 10^{-4} \, \text{s}^{-2} \) and \( U = 15 \, \text{m} \, \text{s}^{-1} \).

Although the warming-induced increase in \( z_1 \) is comparable to that for \( z_2 \), the windward ascent in the three-layer model turns out to be insensitive to that change over the parameter regime of interest. Figure 11 shows the dependence of \( w_L \) at 1.5 km above the center of the windward slope (near the region of maximum orographic condensation) on the other two key parameters, \( z_2 \) and \( N_2 \). The extreme-event mean values for the control and the warmer climate are indicated by the green and red dots, respectively. Despite its limited dynamics, the linear model provides a good estimate of the mean vertical velocity over the eastern slope (compare Fig. 9e and Fig. 11b). Moreover,
the 2.3 % K$^{-1}$ sensitivity of $\delta w_L/w_L$ reasonably approximates the 2.8 % K$^{-1}$ sensitivity of then extreme-event averaged vertical velocity $\delta \tilde{w}/\tilde{w}$ at the same 1.5 km level above the center of the windward slope.

Therefore, the enhanced vertical motion in eastern-slope extreme events occurs because the atmosphere has become better tuned to produce strong mountain waves. One may ask why the western-slope extreme events do not have significant dynamical enhancement. The answer is that the background wind speeds during western-slope events make it much more difficult to produce a similar degree of mountain-wave enhancement. The changes in tropopause height and upper tropospheric static stability for the western-slope events are actually somewhat larger than those for the eastern-slope cases. As the climate warms, $z_2$ increases from 8.7 to 10.2 km, and $N_2$ increases from $1.13 \times 10^{-4}$ and $1.38 \times 10^{-4}$ s$^{-2}$. The sensitivity of $w_L$ to these changes is, nevertheless, much reduced because the cross-mountain winds are much stronger during the western-slope events. The average extreme-event winds are a roughly uniform 15 m s$^{-1}$ in the east, whereas they range from about 20 m s$^{-1}$ at the top of boundary layer to 60 m s$^{-1}$ near the tropopause in the west. Fig. 11a shows how this change in the upstream winds modifies the changes in $w_L$. The computation in Fig. 11a uses the full Klemp and Lilly (1975) linear model including shear such that the winds vary linearly from 20 to 60 m s$^{-1}$ between the ground and the tropopause. The vertical velocities in Fig. 11a are roughly 50% larger than the average values for a point 1.5 km above the surface in Fig. 9b, presumably due to the simplified dynamics in the linear model (neglect of finite-amplitude and three-dimensional effects, no boundary layer or moisture). Nevertheless, they both exhibit almost negligible sensitivities to warming ($\delta w_L/w_L = 0.01 \% \text{ K}^{-1}$, $\delta \tilde{w}/\tilde{w} = -0.05 \% \text{ K}^{-1}$).
6. Elevational Dependence

Extreme orographic precipitation and its sensitivity to global warming both vary with elevation. The fraction of precipitation that falls as snow and its vertical distribution can have a major impact on runoff and flooding (Hamlet and Lettenmaier 2007). For example, hydrological model forecasts for several California watersheds suggest that a 660 m increase in the elevation of the melting level can triple the runoff during 24-hour rainfall events (White et al. 2002).

As shown by the black curves in Fig. 12, the 24-hour extreme-event-averaged precipitation rate \( P \) in the control climate increases with elevation until about \( 0.8h_m \) (a height of 2 km), and the elevation dependence is more pronounced over the western slopes than in the east. Also plotted in Fig. 12 are the column-integrated individual terms in the surface precipitation budget: the condensation rate \( C \), the horizontal flux convergence of rain and snow \( Q_{r/s} \), and the convergence of cloud water and cloud ice \( Q_{w/i} \). When integrated over a 24-hour period, the sources and sinks of the hydrometeors in a column extending from the Earth’s surface to the top of the atmosphere should balance such that

\[
P = C + Q_{w/i} + Q_{r/s}.
\]

In contrast to the precipitation, as shown in Fig. 12, the column integrated condensation is maximized over the lower part of the mountain and decreases as the surface elevation rises. The difference between \( P \) and \( C \) is almost completely accounted for by \( Q_{r/s} \) due to the variations in the downwind transport of rain drops and snow. The magnitude and variation in \( Q_{r/s} \) is greater over the western slopes than in the east because, in the west, more extreme precipitation events occur in wintertime when the upslope westerlies are stronger and there are more slowly falling snow particles available for advection. In contrast to \( C \) and \( Q_{r/s} \), \( Q_{w/i} \) makes only a small contribution to the precipitation budget.
Neglecting the small contribution from $Q_{w/i}$, (12), the sensitivity of the mean column-integrated precipitation rate to increased CO$_2$ satisfies

$$\frac{\delta P}{P} = \frac{\delta C}{P} + \frac{\delta Q_{rl/s}}{P}. \quad (13)$$

The amplitude of each term in (13) is shown in Fig. 13. While the sensitivity of condensation ($\delta C/C$) shows little dependence on surface elevation (not shown), the sensitivity of precipitation ($\delta P/P$) and the contribution of condensation ($\delta C/P$) generally decrease with elevation. This is primarily because the factor $1/P$ decreases with elevation (Fig. 12). For the eastern-slope events, the normalized changes in condensation $\delta C/P$ account for most of the precipitation sensitivity, but over the upper part the western slope events (between elevations of 1.4 and 2.2 km), $\delta P/P$ is much smaller than $\delta C/P$. This difference between $\delta P/P$ and $\delta C/P$ is associated with large negative values of $\delta Q_{rl/s}/P$ over the same upper part of the western slope. These negative values of $\delta Q_{rl/s}/P$ arise almost entirely because the column integrated horizontal flux of snow becomes more divergent above the upper slopes of the mountain in the warmer climate.

Separating the detailed factors responsible for the increased divergence of $Q_{rl/s}$ in the warmer climate is rather involved, but the common denominator is the rise in the melting level. In those extreme events, the melting level in the warmer climate rises by about 1 km in our simulations via processes detailed in Minder et al. (2011). As a consequence, most of the surface precipitation above a height of 500 m switches from snow to rain (Fig. 14), which as noted previously, could produce more runoff and flooding during these extreme events.

7. Summary and discussion

The processes responsible for changes in mid-latitude extreme orographic precipitation in a warmer world have been examined using a hierarchy of models to effectively simulate 40 years
of weather over an idealized north-south ridge on the western margin of a continent very roughly representative of western North American. Control and doubled-CO\textsubscript{2} climates were computed using the GFDL global HiRAM model, and the top ten 24-hour precipitation events on the western and eastern mountain slopes in both climates were re-simulated at higher resolution with the WRF weather prediction model.

The extreme precipitation events on the western and eastern slopes of the idealized mountains tend to occur in different seasons and under different weather patterns. The western-slope events mostly occur in winter months, when a strong atmospheric river embedded in a westerly jet impinges on the western side of the mountains. The eastern-slope events occur most frequently in summertime, when a cyclone to the south produces southeasterly winds and an intense plume of moisture impinges on the slopes of the region experiencing heavy rainfall. Several of the eastern-slope extreme events exhibited synoptic weather patterns strikingly similar to the September 11-13, 2013 floods along the Colorado Front Range.

The 24-hour precipitation intensities for the top-10-event WRF simulations increased on the eastern side by 5.9\%K\textsuperscript{-1} of global-averaged surface temperature increase, and by a somewhat smaller 4.2\%K\textsuperscript{-1} on the western side. Similar values of 6.3\%K\textsuperscript{-1} over the eastern slopes and 3.9\%K\textsuperscript{-1} in the west were obtained for the top 40 events in the HiRAM simulations. The vertically-integrated condensation rate for the top-10-event WRF simulations gave a good approximation to the sensitivity of the precipitation to global warming, including the roughly 2\%K\textsuperscript{-1} difference in the sensitivity between the eastern and western slopes.

The thermodynamic sensitivity of condensation in eastern-slope and western-slope precipitation extremes is estimated as 3.9\%K\textsuperscript{-1} and 5.0\%K\textsuperscript{-1}. These values are close to previous estimates by Shi and Durran (2014) for general orographic precipitation on the windward side of midlatitude mountains under the assumption that the incoming flow is saturated (as in these
extreme events). Under these conditions, the thermodynamic sensitivity ranged between 4 and 5 %K−1 (δΓs/M in their Fig. 11). The thermodynamic sensitivity for the western-slope extreme events here is about 1 %K−1 greater than that for the warmer summertime events over the eastern slope. This is consistent with their wintertime occurrence at colder temperatures.

The difference between the thermodynamic sensitivity of eastern and western slope extremes was overshadowed by a 3 %K−1 greater dynamical contribution to precipitation sensitivity in the east than in the west due to differences in the vertical velocity. The sensitivity of the vertical velocities driving the the eastern- and western-slope events was well approximated by a three-layer linear mountain wave model and is due to warming induced increases in upper tropospheric static stability and the tropopause height. The difference in sensitivity for the eastern- and western-slope vertical velocities is primarily due to differences in the strength of the mean cross-mountain flow, with much weaker winds occurring during the eastern-slope events. Higher precipitation sensitivities might have been expected to occur on the eastern side through an alternate thermodynamic mechanism proposed by Siler and Roe (2014). They noted that under moist saturated conditions, increases in surface temperature produce relatively more condensation aloft, which is more easily advected downwind of the crest to fall on the lee-slope. None of the extreme eastern-slope events in our simulations involve significant westerly flow at the mountain crest, and as a consequence, their downwind-advection mechanism is not active.

Although the precipitation thresholds defining our extreme events shift with the climate to remain “once-per-year” and “once-per-four-year” events, the number of events exceeding the control-climate thresholds increases dramatically in the warmer world. On both sides of the mountain, once-per-year events in the control climate occur on average three times per year in the warmer world. Once-per-four-year events shift, on average, to once every year.
The sensitivity of the precipitation to warming generally decreases with altitude in the WRF simulations, although much of this change is simply due to the way the baseline control-climate precipitation increases with elevation. An exception occurs over the western-slopes on the upper portion of the mountain, where the precipitation sensitivity is further reduced by an increase in the column-averaged divergence of snow and rain. This increased divergence is related to an increase in the height of the melting level in the warmer climate. The most important impact of the change in melting level, however, is to produce a major shift from snow to rain over much of the mountain slope. Such shifts during extreme events have the potential to produce much more runoff and flooding over mountain slopes.

Clearly one must expect quantitative differences between these idealized simulations and climate-model simulations of extreme precipitation over any actual mid-latitude mountain range, and further study is required to determine specific real-world responses. Nevertheless, the general physical basis for the changes in mid-latitude orographic precipitation extremes revealed by our idealized simulations may have widespread applicability. One piece of evidence supporting this comes from Diffenbaugh et al. (2005) and Singh et al. (2013) who, consistent with our result that eastern-slope extreme precipitation increases more rapidly than in the west, found a weakening of the orographic rain shadow in the warmer climate over the north-western United States that was largely due to changes in extreme precipitation.

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<table>
<thead>
<tr>
<th>Threshold</th>
<th>Western Control</th>
<th>Western $2 \times \text{CO}_2$</th>
<th>Eastern Control</th>
<th>Eastern $2 \times \text{CO}_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>105.1 mm</td>
<td>40</td>
<td>128</td>
<td>40</td>
<td>116</td>
</tr>
<tr>
<td>124.8 mm</td>
<td>10</td>
<td>42</td>
<td>10</td>
<td>47</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Scheme</th>
<th>Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microphysics</td>
<td>WSM5</td>
</tr>
<tr>
<td>Longwave radiation</td>
<td>CAM</td>
</tr>
<tr>
<td>Shortwave radiation</td>
<td>CAM</td>
</tr>
<tr>
<td>Surface layer</td>
<td>MM5</td>
</tr>
<tr>
<td>Land surface</td>
<td>Noah</td>
</tr>
<tr>
<td>Planetary boundary layer</td>
<td>YSU</td>
</tr>
<tr>
<td>Cumulus parameterization</td>
<td>Kain-Fritsch</td>
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