Toward retrieving properties of the tenuous atmosphere using space-based lidar measurements

Graeme L. Stephens and Richard J. Engelen
Department of Atmospheric Science, Colorado State University, Fort Collins, Colorado, USA

Mark Vaughan
NASA Langley Research Center, Hampton, Virginia, USA

Theodore L. Anderson
Joint Institute for the Study of the Atmosphere and Oceans, University of Washington, Seattle, Washington, USA

Abstract. This paper introduces a method that can incorporate different information into the lidar retrieval problem as an attempt to address the backscatter–to–extinction ambiguity that plagues the usefulness of lidar backscattering measurements. The approach, suited for application to spaceborne lidar data, inverts the lidar equation via an optimal estimation method. This method is illustrated using three examples drawn from LITE data. Retrievals using only lidar backscatter as input were compared to retrievals performed using an iterative solution to the lidar inversion with the same input. The two methods produced essentially identical results. The new method, however, offers a number of advantages compared to other methods, including (1) the ability to incorporate different kinds of information under a common retrieval philosophy. This feature is illustrated with the formal introduction of optical depth into the lidar inversion. In this paper, optical depth information, derived from the ldar transmission estimates, is combined with backscatter measurements making it possible to retrieve the backscatter–to–extinction ratio in addition to extinction profiles given certain caveats noted in the paper. (2) The method provides a number of ways for evaluating the quality of the retrieval. Notably, the retrieval approach predicts full error diagnostics identifying sources of error due to measurement uncertainty (instrument noise and calibration uncertainty), model error (containing all the assumptions built into the lidar equation and its parameters), as well as a priori error due to the influence of compiled databases on lidar backscatter of aerosol and cloud. When no optical depth information is available, the retrieval errors are largely dominated by the (large) uncertainty attached to the backscatter-to-extinction coefficient k. Under these circumstances the retrievals are only meaningful to the extent that k and its related uncertainty is known. When optical depth is introduced as a form of measurement, the error contributions shift to the extent that retrieval errors become dominated by the measurement error attached to the optical depth itself.

1. Introduction

Tenuous layers of aerosol and thin cirrus clouds introduce important albeit complex effects on the radiation budget of the planet. These effects occur as a consequence of the way aerosol and cloud particles scatter and absorb radiation. These radiative processes, in turn, are determined by optical properties governed by the composition, size, structure, and number density of the particles that form the layers in question. Simply observing the spatial distribution of the more tenuous cloud and aerosol layers is a challenge let alone measuring the complex physical and optical properties.

With the advent of the light detection and ranging (lidar) measurement system emerged a tool sensitive enough to detect the most tenuous scattering layers of the atmosphere. Lidar systems have evolved significantly over the past few decades [e.g., *Measures*, 1984; *Grant*, 1997] being used extensively to detect aerosol [*McCormick et al.*, 1993; *Wandinger et al.*, 1995; *Jager*, 1991; *Sassen and Horel*, 1990] as well as cirrus clouds [e.g., *Platt*, 1979; *Platt et al.*, 2001; *Sassen*, 1991; *Wylie et al.*, 1995; *Spinhirne and Hart*, 1990]. The era of spaceborne lidar systems is now upon us with a flight of the backscatter lidar as part of the Lidar in Space Technology Experiment (LITE) flown in 1994 on the space shuttle [*Winker et al.*, 1996]. LITE demonstrated the capability of lidar in detecting cloud and aerosol, giving an unprecedented view of the vertical structure of the scattering atmosphere and is to be followed with the launch of the GLAS later in 2001 and the lidar of PICASSO-CENA (P-C) [*Winker and Wielicki*, 1999] in 2004.

Although the backscattering measurements by ground, airborne, and eventually spaceborne lidar systems promise much, actual retrieval of particle optical property information using lidar alone has proven to be difficult. The difficulty arises from an intrinsic ambiguity that occurs between the effects of backscattering and extinction that arises from a combination of scattering and absorption. For example, consider two volumes containing different numbers of scatterers buried somewhere deep in a cloud or aerosol layer. The volume with a greater number of scatterers does not necessarily produce a signal larger than that derived from the volume of fewer particles since the increased contribution by the greater number of backscattering in that volume can be offset by a simultaneous decreased signal associated with attenuation within the same volume. These competing effects cannot be separated from measurements of lidar backscatter alone, and other information is required to address this so-called backscatter-to-extinction ambiguity. While more advanced lidar systems, such as the Rayleigh lidar system [e.g., *Grund and Eloranta*, 1991] and Raman lidar systems [*Ansmann et al.*, 1990] provide a way of unraveling this ambiguity, retrievals based on data from the simpler backscattering systems planned for space in the coming years have to confront this problem. More obvious approaches to addressing this problem either introduce an explicit relationship between the backscattering and the extinction and take this to be known a priori or alternatively introduce other forms of measurement that can be related independently to extinction (or backscatter) as exemplified by the method of *Fernald et al.* [1972], the LIRAD method of *Platt* [1979], and others.

The purpose of this paper is primarily to outline a method to retrieve profiles of particle extinction from measured lidar backscattering profiles. The focus of attention is on application of the retrieval algorithm to the spaceborne lidar data of LITE. The challenge is to identify ways to constrain, quantitatively, the backscatter–to–extinction properties and related uncertainty whether from some independent source of knowledge about the scattering of the particles such as typing aerosol by air mass or cloud by temperature or from other measurements containing independent information about particle extinction. Once identified, a method has to be developed that can appropriately add this alternate heterogeneous knowledge into a quantitative retrieval method. The essential purpose of this paper is to introduce one such retrieval approach not previously applied to lidar retrievals. The method is developed and illustrated using examples drawn from LITE data, and the additional information needed to constrain the retrieval problem is introduced in the form of optical depth.

The main body of the paper begins with a brief introduction to LITE and presents orbit data selected for analysis. Section 3 then follows with a brief outline of the lidar retrieval problem and offers information about the backscatter-to-extinction characteristic of aerosol and cirrus clouds. A new lidar retrieval method is introduced in section 4 followed by examples of retrievals in section 5 which, as in past work [e.g., *Platt*, 1979], reveal an accute sensitivity to the assumed value of the backscatter–to–extinction ratio and its related uncertainty. We show how poor knowledge of this parameter leads to an increasingly unacceptable range of solutions particularly as the optical depth of the layer increases. Both section 6 and the Appendix outline approaches that introduce optical depth in the retrieval either as a form of measurement (section 6) or as a contraint (Appendix A). The results of the paper along with conclusions derived from these results are summarized in the final section of the paper.

2. Lidar Profile Examples From LITE

The Lidar in Space Technology Experiment (LITE) was flown aboard the space shuttle Discovery during September 1994. As the first civilian effort to operate an active instrument on space platforms for environmental research purposes, LITE provided near-nadir profiles of backscatter along the orbit track using a three-wavelength doubled and tripled Nd-YAG 1064 nm lidar. For the most part, independent retrievals of cloud and aerosol properties were not available to match these lidar profiles. Furthermore, the lidar backscatter in the presence of clouds typically saturated the detector when set in high gain aerosol mode and the noise introduced by scattered sunlight degraded the quality of the data to disallow Rayleigh and aerosol scatter measurements during the daytime portions of the orbits. In this study we choose three LITE profiles to introduce the retrieval method. Two of the profiles were chosen because it was possible to determine the two-way transmittance through a thin cirrus layer in one case and a lofted aerosol layer in the second case, thereby providing some estimate of optical depth of each respective layer. This transmission approach is frequently used in analyses of lidar data [see *Young*, 1995].

In this study, only nighttime portions of orbit data are analyzed and then only three specific backscattering profiles are used to examine the lidar retrieval problem. Data from segments of the two orbits are presented as images of backscattering profiles measured along the orbit track of the shuttle. The top panel of Plate 1 is an image of an approximate 350 km section of the nighttime portion of the LITE orbit 129 located in the vicinity of the Sudan-Ethiopia border. The backscattering profiles composited to create this image reveal the presence of two layers of thin cirrus clouds overlying a lower more extensive layer of aerosol. The cirrus layers are less extensive than the aerosol layer with one very thin layer above 15 km and a lower layer below 15 km is dense enough in portions to saturate the returned signal. The aerosol layer is located below about 5 km being lofted off the surface for most of the section of the orbit. The source of this aerosol is presumably wind-blown dust from neighboring desert regions. The bottom image of Plate 1 is a portion of orbit 147 showing an extensive layer of aerosol generally lofted above the surface and located between approximately 1-6 km.

Plate 1.

Figure 1 presents individual backscattering profiles extracted from a 20 shot average, equating to an approximate 15 km along track average, at the locations (10.3°N, 31.7633°E) along orbit 129 and (20°N, 22°W) of orbit 147, both locations being indicated in Plate 1 by the vertical dashed line. The estimated optical depths of the cirrus layer (orbit 129) and aerosol layer (orbit 147) are quoted in the respective profiles of Figure 1. These values of optical depths do not apply to the entire profile but to that part of the raw profile that can be identified as the layer in question. The profile data are also averaged in the vertical with the data shown corresponding to a 60 m vertical resolution. The profile extracted from orbit 129 corresponds to the case of a very thin layer of cirrus over a relatively deep aerosol layer. The second profile (orbit 147) corresponds to a lofted layer of aerosol located roughly between 1.5 km and 5.5 km above the surface.

Figure 1.

3. Lidar Retrieval Problem

The lidar equation governs the relation between the range-resolved measured backscattering power $P(R)$ and the scattering and attenuation properties of the atmosphere. This equation may be written as follows:

$$\ln(C \times P(R)R^2) \approx \ln[\beta_{\text{Ray}}(R) + \beta(R)] - 2\int_0^R [\eta(r')\sigma(r') + \sigma_{\text{Ray}}(r')]dr' \;, \tag{1a}$$

where $P(R)$ is the raw backscatter at range R, $\beta(R)$ is the backscattering due to aerosol or cloud, $\beta_{\text{Ray}}(R)$ is the Rayleigh-backscattering coefficient, $\sigma(R)$ is the particle extinction coefficient, $\sigma_{\text{Ray}}(R)$ is the Rayleigh extinction, C is a factor that represents instrument calibration, and η is a factor that accounts for multiple-scattering processes. This calibration factor can be derived by matching portions of the measured backscattering profile taken to arise from pure molecular scattering. Figure 2 is an example of such a calibration, showing the profile of raw lidar data P from orbit 147 analyzed in this study plotted against the calculated Rayleigh-backscattering coefficient derived using a temperature profile matched to that portion of the orbit. The source of this temperature profile is from weather forecasts carried out for the period of LITE by the European Centre for Medium-Range Weather Forecasts. The top portion of the profile (corresponding to the smaller values of backscatter) represents the pure Rayleigh-scattering portion of the profile and a simple linear fit, as indicated, directly provides C and associated offset values that must be incorporated into (1a). The portions of the profile which deviate from this linear fit correspond to the aerosol layer noted above.

Figure 2.

Equation (1a) is a parametric form of the equation of transfer that represents lidar propagation in a scattering medium. Multiple-scattering processes are represented by the factor $\eta < 1$, which reduces the attenuation of the lidar beam to account for the effect of multiple scattering on the propagation. Unless stated otherwise, we hereinafter assume that $\eta = 1$ although we note that multiple scattering of the spaceborne lidar specifically and surface lidar more generally continues to be a topic of study [e.g., *Miller and Stephens*, 1999; *Bissonnette et al.*, 1995] and is an additional source of retrieval uncertainty that is not specifically included in the analyses presented below.

An equivalent form of (1a) is

$$\ln(C \times P(R)R^2) \approx \ln[\beta_{\text{Ray}}(R) + k(R)\sigma(R)] - 2\int_0^R [\eta(r')\sigma(r') + \sigma_{\text{Ray}}(r')]dr' \;, \tag{1b}$$

where the cloud and aerosol backscatter coefficient $\beta(R)$ of (1a) is replaced by the product $k(R)\sigma(R)$, where $k(R)$ is the backscatter-to-extinction coefficient, hereinafter considered independent of range and discussed further below. The goal is to invert (1a) or equivalently (1b) to obtain either the profile of particle extinction, namely $\sigma(R)$, or the profile of particle backscatter $\beta(R)$. The inversion can follow two equivalent paths. One path inverts (1a) for $\beta(R)$ given the replacement of $\sigma(r')$ by $\beta(r')/k$ in the attenuation term. The particle extinction profile then follows as the product $\beta(R)/k$. The second approach inverts (1b) for $\sigma(R)$ and $\beta(R)$ subsequently follows as the product $k\sigma(R)$.

The key factor that facilitates the inversion of the lidar equation either in terms of $\beta(R)$ or $\sigma(R)$ is the backscattering-to-extinction ratio k and its enabling role in the inversion of (1a) and (1b) is well known [e.g., *Stephens*, 1994]. Unfortunately, this parameter is highly variable depending on the size, shape and composition of the scattering particles. Table 1, adapted from *Anderson et al.* [2000], provides measurements or estimates of this ratio for various aerosol types. The table lists values of the so-called lidar ratio $\mathcal{S}$,

Table 1.

$$\mathcal{S} = \frac{4\pi}{k} \,, \tag{2}$$

collated from various aerosol data sources on the basis of different measurement approaches. While the actual volume of data for tropospheric aerosol, as well as for cirrus clouds as described below in relation to Figure 3, is limited, the available data do allow us to place bounds on the values of k. For example, the values listed in Table 1 range from about $\mathcal{S}$ = 15 sr to 91 sr where this range is broadly governed by the aerosol mode although the ratios also vary significantly within these modes. Also noteworhty are the values for Hawaii, indicating that this ratio is reasonably well constrained for this particular case where the sampling was conducted in a very consistent (maritime) air mass. Typical values of k ($\mathcal{S}$) for the coarse mode range between 1.26 and 0.314 (10 to 40 sr), while values for submicron pollution aerosol particles range between 0.314 and 0.14 (40 to 90 sr) [*Ackerman*, 1998; *Muller et al.*, 1998; *Anderson et al.*, 2000; *Reagan et al.*, 1988]. In this study we do not attempt to represent comprehensively this variability but rather exemplify properties of coarse mode (seasalt and dust) and accumulation mode (dirty and clean pollution) aerosol types by considering a range of values of $\mathcal{S}$ between 20 and 80 sr and k between 0.628 and 0.157.

Figure 3.

Values of the backscatter-to-extinction ratio of cirrus clouds also vary significantly. Figure 3, adapted from the work of *Platt et al.* [2001], indicates that k varies by a factor of almost 4 for the tropical cirrus clouds analyzed using the LIRAD measurement technique applied to the lidar data. The apparent temperature dependence suggests that it may be possible to restrict the range of k for cirrus by assuming this parameter varies as a function of midcloud temperature as implied by the results of Figure 3.

The lidar retrieval problem considered in this paper seeks to invert (1) to obtain profiles of σ. There are a number of published techniques for inverting the lidar equation, including the Klett method [*Klett*, 1981] for a single-component system, and the methods of *Fernald* [1984] and the iteration methods of *Platt* [1979] and *Alvarez and Vaughan* [1994] for a two-componenet system considered in this paper. Since measurements of k (and $\mathcal{S}$) for ice crystal clouds and aerosol have been limited to a few surface and ground-based lidar sites worldwide [*Anderson et al.*, 2000], and since the range of variability of this parameter is known to be large, a retrieval algorithm applied to global spaceborne lidar data by necessity has to assign a large uncertainty to this parameter, thus compromising the quality of the information returned by the algorithm. The challenge is to identify ways to constrain the range of k, thus reducing the uncertainty attached to this parameter, thereby improving the overall quality of the retrieval. This constraint could be derived from some independent source of knowledge about the scattering of the particles such as typing aerosol by air mass, or gross information about particle size, cloud by temperature, or from other measurements containing independent information on particle extinction. Once identified, a method has to be developed that can appropriately add this alternate heterogeneous knowledge into a quantitative retrieval method. The essential purpose of this paper is to introduce an approach that is capable of being extended to incorporate this kind of knowledge in a variety of forms with the lidar measurements.

4. New Lidar Algorithm

The approach developed here to invert the lidar equation is based on the optimal estimation method popularized by the work of *Rodgers* [1976, 1990] among others. This approach offers a number of advantages over the more usual lidar retrieval methods introduced above. A key goal of this paper is to illustrate the primary advantages of this approach, which include (1) an ability to incorporate multiple sources of heterogeneous information under a common retrieval philosophy. These sources of information can extend beyond the 532 nm lidar backscatter of the spaceborne lidar. For example, the information could come from lidar measurements at an additional wavelength (such as at 1064 nm as in LITE and also proposed by PICASSO-CENA), or be introduced in the form of optical depth derived from some other independent source such as IR radiances as in the example of the LIRAD method [*Platt*, 1979] or the information might come in the form of scene classification pointing to specific sets of parameters among the many possibilities. (2) Data available to the retrieval can also vary along the orbit due to, for example, day–night orbit differences among other factors. The same algorithm can process different types and amounts of data accordingly. (3) Retrieval diagnostics in the form of detailed error budgets, including a breakdown of error components and metrics of information content follow directly from the method. This comprehensive set of diagnostics provides any user of the data clear measures of the retrieval quality and the extent of reliance on other extraneous data.

4.1. General Optimal-Estimation Approach

The lidar-observing system, mathematically expressed in terms of the lidar equation (1), can be generally written in an alternate, conceptual form

$$\mathbf{y} = F(\mathbf{x}, \mathbf{b}) + \varepsilon_y \; , \tag{3}$$

where $\mathbf{y}$ is a vector of the measurement variable (backscatter power) and ε_y is the measurement error (ostensibly arising from calibration of the lidar); F is the forward function, $\mathbf{x}$ is a vector of the true extinction profile (as distinct from $\hat{\mathbf{x}}$, which is our (imperfect) retrieval of $\mathbf{x}$), and $\mathbf{b}$ is a vector of "model" parameters that are not actually retrieved but fundamentally govern the relation between the measurement and the retrieved parameter of interest. In reality, both the forward model and the model parameters must be approximated, producing

$$\mathbf{y} = f(\hat{\mathbf{x}}, \hat{\mathbf{b}}) + \varepsilon_y + \varepsilon_f \; , \tag{4}$$

where ε_f now reflects an additional source of uncertainty associated with the forward function and assumed values of $\hat{\mathbf{b}}$ as discussed below.

We follow the notation of *Rodgers* [1990] and introduce, formally, the retrieval scheme as

$$\hat{\mathbf{x}} = I(\mathbf{y}, \hat{\mathbf{b}}, \mathbf{x}_a, c) \; , \tag{5}$$

which is the "inverse model" requiring some a priori information $\mathbf{x}_a$ and other information c that is not formally part of the forward function but nevertheless may be needed by the inversion process.

The optimal estimation approach constructs $\hat{\mathbf{x}}$, using the forward model to obtain the most likely solution consistent with both the measurements and any given a priori knowledge of $\mathbf{x}$. Under the assumption of Gaussian statistics, the optimal solution is found by minimizing the cost function

$$\begin{aligned} \Phi &= (\hat{\mathbf{x}} - \mathbf{x}_a)^T \mathbf{S}_a^{-1} (\hat{\mathbf{x}} - \mathbf{x}_a) \\ &\quad + (\mathbf{y} - f(\hat{\mathbf{x}}, \hat{\mathbf{b}}))^T \mathbf{S}_y^{-1} (\mathbf{y} - f(\hat{\mathbf{x}}, \hat{\mathbf{b}})) + C(\hat{\mathbf{x}}) \; , \end{aligned} \tag{6}$$

with respect to $\hat{\mathbf{x}}$. A brief note about the assumption of Gaussian statistics is warranted. The optimal estimation approach introduced here is an implementation of the Bayes theorem and the details of how one goes about estimating the most probable solution varies according to the details of the form of the probabilities assumed. In principle, any non-Gaussian statitistics could be invoked (this would change the details of the solution presented below in significant ways). E. T. Jaynes (*Probability Theory: The Logic of Science, 1996.* unpublished manuscript, 1996 (book available at http://omega.math.albany.edu:8008/JaynesBook.html)) notes that according to the principle of maximum entropy, the Gaussian distribution is the most appropriate if only a mean and variance is known. Alternative distributions, unless known and rigorously justifiable, add spurious information to the retrieval and biases the estimatation of the most probable solution.

The first term of (6) represents that part of the cost function associated with constraints imposed by a priori data. The degree to which this constraint is to be applied is ultimately determined by the covariance matrix $\mathbf{S}_a$, which contains measures of the confidence placed in $\hat{\mathbf{x}}_a$. The second term represents that part of the cost function defined by matching the measurement $\mathbf{y}$ to the model $f(\hat{\mathbf{x}}, \hat{\mathbf{b}})$. Minimizing the cost function defined by this term alone is equivalent to a weighted least squares estimation [e.g., *Menke*, 1989], where the weighting in this sense is established through the covariance error matrix $\mathbf{S}_y$. The third term of this particular cost function represents an additional constraint that might be imposed from some other form of information about $\mathbf{x}$, thus emphasizing how such information can be added in the retrieval as desired. We describe, in Appendix A, an example of the inversion of the lidar equation for which $C(\hat{\mathbf{x}})$ is in the form of an integral constraint expressed in terms of the optical depth.

Ignoring the final term of (6) for now, the solution $\hat{\mathbf{x}}$ that lies at the minimum of the cost function is [e.g., *Marks and Rodgers*, 1993]

$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{S}_a \mathbf{K}^T \mathbf{S}_y^{-1} [\mathbf{y} - f(\hat{\mathbf{x}})] \;, \tag{7}$$

where $\mathbf{K}^T$ is the transpose of the weighting function matrix constructed of $\partial f(\hat{x}_i)/\partial(\hat{x}_j)$. In the case of the lidar problem, the weighting function matrix is composed of elements representing the sensitivity of the forward model evaluated at range R_i to changes in the extinction at range R_j.

The specific form of the covariance matrices and the weighting function matrix, as applied to the lidar problem, is discussed in section 4.2, but some general comments about $\mathbf{S}_y$ are warranted at this stage. Following *Marks and Rodgers* [1993], we consider $\mathbf{S}_y$ to contain two general sources of error, namely,

$$\mathbf{S}_y = \mathbf{S}_\varepsilon + \mathbf{S}_f \;, \tag{8}$$

where the first term of the right-hand side of (8) is the error attributed to the measurement of $\mathbf{y}$ (i.e., ε_y), and the second term contains the errors that can be attached to the forward model (ε_f). The latter, in turn, can be expressed in terms of the quantity

$$\Delta f = F(\mathbf{x}, \mathbf{b}) - f(\mathbf{x}, \mathbf{b}) - \frac{\partial f}{\partial \mathbf{b}}(\hat{\mathbf{b}} - \mathbf{b}) \ , \qquad (9)$$

where the difference term defined by the first two factors accounts for biases that might exist in the forward model. The third term is the error associated with uncertainties in model parameters. Quantifying bias errors is difficult in general, and these are hereinafter ignored although bias errors will have to be addressed when applying the algorithm to real data. With this assumption, it thus follows that [refer to *Marks and Rodgers*, 1993]

$$S_f = \mathbf{K}_b^T \mathbf{S}_b \mathbf{K}_b \ ,$$

where $\mathbf{K}_b$ is the sensitivity matrix determined by the factors $\partial f(\hat{x}_i, \mathbf{b})/\partial \mathbf{b}$, and $\mathbf{S}_b$ is the error covariance matrix formed by the errors attached to the model parameters $\mathbf{b}$.

Given suitable specification of all error terms and weighting functions, the solution of (6) proceeds by iteration [*Marks and Rodgers*, 1993],

$$\hat{\mathbf{x}}^{n+1} = \mathbf{S}_x^n [(\mathbf{S}_a^{-1} \mathbf{x}_a + \mathbf{K}^{nT} \mathbf{S}_y^{-1} [\mathbf{y} - f(\hat{\mathbf{x}}^n) + \mathbf{K}^n (\hat{\mathbf{x}}^n)]] \ , \qquad (10)$$

where $\mathbf{S}_x^n$ is the error covariance matrix of the nth iteration

$$\mathbf{S}_x^n = (\mathbf{S}_a^{-1} + \mathbf{K}^{nT} \mathbf{S}_y^{-1} \mathbf{K}^n)^{-1} \ . \qquad (11)$$

Convergence of this iteration is achieved by the test

$$\Delta \hat{\mathbf{x}}^T S_x^{-1} \Delta \hat{\mathbf{x}} \ll n_x \ , \qquad (12)$$

where n_x is the dimension of the $\hat{\mathbf{x}}$ vector.

The retrieval error covariance matrix $\mathbf{S}_x$ can also be written as

$$\mathbf{S}_x^n = \mathbf{D}_y \mathbf{S}_y \mathbf{D}_y^T + \mathbf{D}_a \mathbf{S}_a \mathbf{D}_a^T \qquad (13)$$

where

$$\mathbf{D}_y = \mathbf{S}_x \mathbf{K}^n \mathbf{S}_y^{-1} \ , \qquad (14)$$

$$\mathbf{D}_a = \mathbf{S}_x \mathbf{S}_a^{-1} \ . \qquad (15)$$

The advantage of this particular form of error covariance is that the individual components that contribute to the total error can be quantified. In this study we consider three main sources of retrieval error for discussion, an a priori error defined by the second term of the right-hand side of (13), the measurement error representing the contribution $\mathbf{S}_\varepsilon$ in the first term of the right-hand side of (13), and the model error determined by (9) when substituted into (8) and, in turn, into (13).

In addition to the error analysis offered by the approach, other useful diagnostic information can be derived. Goodness-of-fit of the retrieved information to the measurements (including a priori data) can be quantified using a χ^2 test, where

$$\chi^2 = (\hat{\mathbf{x}} - \mathbf{x}_a)^T \mathbf{S}_a^{-1} (\hat{\mathbf{x}} - \mathbf{x}_a) + (\mathbf{y} - f(\hat{\mathbf{x}}, \hat{\mathbf{b}}))^T \mathbf{S}_y^{-1} (\mathbf{y} - f(\hat{\mathbf{x}}, \hat{\mathbf{b}}))$$

follows a χ^2 distribution with n_y degrees of freedom (n_y is the dimension of the measurement vector $\mathbf{y}$. According to *Marks and Rodgers* [1993], $\chi^2 \approx n_y$ is typical of a good retrieval and is also typical of the retrievals provided below (although χ^2 statistics are not presented).

The model resolution matrix [e.g., *Menke*, 1989; *Rodgers*, 1990]

$$\mathbf{A} = \mathbf{D}_y \mathbf{K} \tag{16}$$

provides a measure of the dependence of $\hat{\mathbf{x}}$ on the true profile $\hat{\mathbf{x}}$ and thus offers insight on the extent the measurements $\mathbf{y}$ contribute to the retrieved $\hat{\mathbf{x}}$ versus how much a priori information contributes to the retrieval. For an ideal observing system, which is one solely dependent on measurements, $\mathbf{A}$ is the identity matrix. For a system that delivers a retrieval based entirely on a priori input, $\mathbf{A}$ is the zero matrix. For the retrievals described below, very little weight is placed on a priori input resulting in an $\mathbf{A}$ matrix that very closely approximates the desired unity matrix. (For an interpretation of the $\mathbf{A}$ matrix applied to actual data, refer to *Engelen and Stephens* [1999]).

4.2. Application to the Lidar Retrieval Problem

The lidar retrieval problem (1) can be expressed in the form of (3) with the following definitions:

$$\mathbf{x} = \{\sigma(R_j); j = 1, \ldots, N\}\ , \tag{17a}$$

$$\mathbf{y} = \{\ln(C \times P(R_j) R^2); j = 1, \ldots N\}\ , \tag{17b}$$

$$f(\hat{x_j}, \hat{b_j}) = \ln(\beta_{\text{Ray}}(R_j) + k(R_j)\hat{x}(R_j)) - 2\Sigma_{\ell=1}^{j}[c\bar{\beta}_{\text{Ray},\ell} + \eta \bar{x_\ell}]\Delta R\ , \tag{17c}$$

defined at each range $R_j, j = 2, \ldots N$ counted from the top of the atmosphere downward where ΔR is the range resolution of the lidar system, and the overbar refers to layer mean values ($\bar{x}_\ell$ is the layer mean extinction ($\bar{x}_j = 0.5(x_j + x_{j-1})$)). We consider two forms of the model parameter vector, one vector being composed of three model parameters that are each a function of range R_j

$$\hat{b} = \{b_1, b_2, b_3\}\ ,$$

where

$$b_1 = \{\beta_{\text{Ray},j}; j = 1, \ldots N\}\ , \tag{18a}$$

$$b_2 = \{k; j = 1, \ldots N\}\ , \tag{18b}$$

$$b_3 = \{\eta_j; j = 1, \ldots N\}\ , \tag{18c}$$

where we have assumed that k is constant throughout the scattering layer under investigation. The second version of the retrieval defines

$$\hat{b} = \{b_1, b_3\}$$

and includes k in the retrieval vector $\hat{\mathbf{x}}$. We defer discussion of this second approach to section 6.

The elements of the weighting function matrix, K_{ij}, constructed by differentiating (17c) with respect to x, follow as

$$\begin{aligned} K_{ij} &= 0 \quad i < j \ , \\ K_{ij} &= -2\eta\Delta R \quad i > j \ , \\ K_{ij} &= \frac{k}{\beta_{\mathrm{Ray},i} + kx_i} - 2\eta\Delta R \quad i = j \ , \end{aligned} \tag{19}$$

and all covariance matrices are assumed to be diagonal with the diagonal elements specified by

$$S_{a,ii} = \sigma_a^2 \ ,$$

$$S_{y,ii} = \varepsilon_y^2 + \sigma_{b1}^2 + \sigma_{b2}^2 + \sigma_{b3}^2 \ ,$$

where ε_y is the error in the measurement and, for both simplicity and for the sake of illustration, this is taken to be 5% of the measured backscatter power, and the remaining error factors follow from (9) and (18b) with

$$\begin{aligned} \sigma_{b1} &= \frac{0.02\beta_{\mathrm{Ray},i}}{\beta_{\mathrm{Ray},i} + kx_i} \ , \\ \sigma_{b2} &= \frac{\Delta k_i x_i}{\beta_{\mathrm{Ray},i} + kx_i} \ , \end{aligned} \tag{20}$$

which, respectively, reflect a 2% error in the specification of the Rayleigh backscatter taken to be typical of the error in estimating Rayleigh backscater from a given temperature profile and an error in k dictated by Δk. Hereinafter, $\sigma_{b3} = 0$ since we consider only the case of $\eta = 1$.

The a priori $\mathbf{x}_a$ applied to (10), used to establish the error according to (20) and further used to begin the iteration, is derived from a simple one step of the interaction solution of the lidar equation (1b) expressed as

$$x_{a,i} = [\exp(y_i + 2\Sigma_{\ell=1}^{i-1}[c\beta_{\mathrm{Ray},\ell} + \eta x_{a,\ell}]\Delta R) - \beta_{\mathrm{Ray},i}]/k \ . \tag{21}$$

We hereinafter set $\sigma_a = 5S_{y,ii}$, reflecting a state of little confidence in a priori knowledge (for example, there is no established climatology of aerosol extinction that can be used to define $\mathbf{x_a}$). Although arbitrary, this has the sought after effect of downgrading the contribution of the a priori information relative to the information contained in the measurement.

5. Aerosol Retrieval Results for Orbit 129

Aerosol extinction profiles were retrieved from the backscattering profile corresponding to the lower portion of orbit 129. It was not possible to infer the optical depth of this lower aerosol layer using the previously mentioned transmission method so the retrieval results introduced below are presented merely to highlight selected features of the lidar retrieval problem. Retrievals are presented for the following values $k = 0.628, 0.314, 0.209$, and 0.157 (or equivalently $\mathcal{S}$ =20,40,60, and 80), broadly encompassing the expected range of values noted in Table 1. The retrieved profiles corresponding to each of these values of k, presented in Figure 4, were derived assuming an error in k of 50%, reflecting a relatively poor knowledge of this parameter (but not so poor as to accommodate a factor of 5 range in variability noted in Table 1). For comparison purposes, results were obtained using an algorithm based on a simple iteration method (hereinafter referred to as the iteration method). The agreement between the two types of retrieval methods is excellent. Since there is no companion information about the physical and chemical characteristics of the aerosol observed during LITE, it is not possible a priori to specify which of the four values of k (or any value within the range considered) is the correct retrieval without considerable uncertainty. Lacking additional information that might reduce the range of uncertainty in k, any one of the four profiles presented in Figure 4 represents a legitimate retrieval solution for the given backscattering profile.

Figure 4.

Table 2 lists the optical depths obtained from the integration of the individual extinction profiles retrieved through the depth of the aerosol layer. The optical depths derived from the retrievals from the iteration method are given in parentheses. The results emphasize an obvious correlation between the optical depth obtained from the integration of the extinction profiles and the value of k assumed in the retrieval. This correlation is a consequence from the fact that a layer of aerosol characterized by low values of k (typical but not exclusive to aborbing aerosol) requires a greater number density to produce the same backscatter compared to the case of an aerosol of high k (such as a purely scattering aerosol). Unfortunately, the retrieved profile not only does not simply scale in a proportional linear way to the value of k but also changes in a nonlinear fashion as clearly seen, for example, when comparing the k= 0.628 and 0.157 profiles of Figure 4. The nonlinear dependence of the retrieval on k is similarly reported by *Platt* [1979].

Table 2.

Figures 5a and 5b present the error budgets associated with two of the profiles presented in Figure 4 ($k = 0.628$ and $k = 0.157$, respectively). Shown are the profiles of extinction as given in Figure 4 and the total error,

Figure 5.

$$\mathrm{error} = \sqrt{S_{x,ii}}$$

(left panel) as well as the contribution of the three individual components (right panels) where the sum of the squares of the componenet errors is equal to the square of the total error. The total error varies through the profile and approaches about 50% as expected since the total error is dominated by the model error which, in this case, is principally determined by the uncertainty in the model parameter k. Since the uncertainty placed on k was assumed to be constant throughout the layer, this resulting model error is approximately uniformly distributed throughout the layer. The error that arises from the influence of the a priori on the retrieval is small despite the fact that the uncertainty assumed for the a priori is large. This too is understandable given that by design, very little of the a priori $\mathbf{x}_a$ actually contributes to retrieval and thus to the error budget.

The results presented above emphasize an important feature of the lidar retrieval problem that although known, warrants further comment. The results underscore the extreme sensitivity of the retrieval to the assumed value of k. As a consequence, large uncertainty in k, typical of what is expected for observations collected over the range of global conditions without any way to constrain this parameter, corresponds to an unacceptably wide range of retrieval solutions. There are two possible ways around this unsatisfactory result. One approach is to provide direct information about k or correlative information that points to specific values of k with reduced uncertainty. Since direct measurement of k has been restricted to a few surface sites worldwide and the amount of this type of data cannot be expected to increase significantly, this strategy is perhaps more appropriately limited to validation of the global retrievals at those limited sites. A second approach is to introduce some form of constraint information to limit the range of values of k using information more readily available to match lidar orbit data. Table 2 supports the idea that a possible source of such information is the optical depth of the layer. Since the optical depth information retrieved using present satellite systems corresponds only to the column optical depth, this approach only works when the optical depth can properly be assigned to the scattering layer being profiled. As we indicated above, it is also possible to determine the optical depth of thin layers using lidar data. This method requires some way of characterizing the return below the layer in question (such as from a sufficiently deep Rayleigh-scattering layer) and cannot be expected to work for the important cases of boundary layer aerosol.

Two different ways of introducing optical depth information are introduced in this paper. The method described in section 6 introduces the optical depth into the retrieval framework in a way that permits the retrieval of k. This approach is especially suited to the backscatter lidar problem characterized by highly uncertain backscatter–to–extinction relationships. A second approach is introduced in Appendix A and uses the optical depth as a form of internal constraint applied to the solution. Although this second approach is applied to the lidar problem in Appendix A, it is better suited to those problems for which the backscatter–to–extinction relation is better known, such as for the problem of radar precipitation retrieval [*L'Ecuyer and Stephens*, 2001].

6. Optical Depth As a Form of Additional Measurement

We now introduce an approach that makes use of the optical depth as a form of measurement with the intention to retrieve both k and the extinction profile directly. The advantage is that a retrieval of k eliminates the unsatisfactory dependence of the lidar retrieval on this parameter and thus removes the major source of error from the retrieval problem. However, it is necessary to assume that k is vertically uniform through the cloud or aerosol. This assumption is questionable and some idea of this limitation, along with associated error, will eventually be required if the algorithm described in this paper is to be applied to real data in quasi-operational mode. The second advantage of the approach described below is that retrieved values of k contain useful information about aerosol.

The approach to introduce optical depth deviates from that described in section 4 only in minor ways now described. The optical depth is added to the measurement vector $\mathbf{y}$ which is now a vector of length $N+1$. The retrieval vector likewise is now extended by one element, where now $x(N+1) = k$. The forward model (optical depth could also be included in a more comprehensive way by extending the forward model to deal with radiance measurements, for example, thus directly integrating the retrieval of optical depth with the lidar retrieval), representing the additional measurement, simply follows as

$$f(x_{N+1}) = \Sigma_{\ell=2}^{N} \bar{x}_\ell \Delta R \; . \tag{22}$$

The weighting function matrix $\mathbf{K}$ is now an $N+1 \times N+1$ matrix where the $N \times N$ block follows from (18) with k replaced by $x(N+1)$. The additional row and column entries are defined thus:

$$K_{i,N+1} = \frac{x_i}{\beta_{\mathrm{Ray},i} + x(N+1)x_i} \; , \tag{23}$$

for $i = 1, \ldots, N$ and

$$K_{N+1,j} = \Delta R \; , \tag{24}$$

for $j = 1, \ldots, N$ ($K_{N+1,N+1} = 0$). The dimensions of the square, diagonal covariance matrices are similarly $N+1$

with the additional diagonal elements defined as

$$S_{a,N+1,N+1} = \sigma_k^2 \, , \tag{25}$$

$$S_{y,N+1,N+1} = \sigma_\tau^2 \, , \tag{26}$$

where σ_k^2 is the uncertainty assigned to the a priori guess for k taken to be large (100%). The error associated with the optical depth, as above, is specified by $\sigma_\tau = 0.05\tau$, representing a 5% error in this quantity as noted above.

Figure 6.

Figure 6a and 6b present the results of this scheme applied to both the cirrus layer observed along orbit 129 and the elevated aerosol layer observed along orbit 147, respectively. The optical depths obtained from the lidar transmission method for each layer, noted previously in reference to Figure 1, were applied in the retrievals shown in Figure 6. Also shown for comparison in the left side of each figure are the equivalent extinction profiles obtained using the iteration method with a value of k deduced from the ratio of the integrated backscatter and optical depth. The profiles of extinction and the values of k retrieved by both methods are closely matched. The right-hand panels of each figure present the component errors which combine to produce the total error presented in the left-hand panels. The differences between the error properties of the retrievals of Figure 6 based on the combination of backscatter measurements and optical depth information and the errors presented in Figure 5 are substantial. Firstly, the total retrieval errors are substantially reduced over the retrievals conducted without optical depth information (e.g., Figure 5). This result should be considered, however, within the context of the assumptions invoked to produce the results of Figure 6, specifically that the backscatter–to–extinction properties are vertically invariant. Secondly, the error budget is now dominated not by model errors but by measurement errors that chiefly reflect the assumed error in optical depth assigned to the transmission method used to derive this information.

7. Discussion and Conclusions

Lidar measurements of the atmosphere are entering a new realm with the launch of simple backscattering lidar systems on a series of upcoming satellite experiments. Conversion of lidar backscattering measurements to relevant information about particles by inversion of the lidar equation is, however, complicated by ambiguities that arise from the need to separate the effects of backscattering from extinction. This leads to inversions that are largely determined by assumptions dealing with the connections between backscattering and extinction. The large range of possible behavior of backscatter–to– extinction together with the overreliance of the end retrieval product on these assumptions is well known and is especially undesirable in the context of the spaceborne measurements that will observe the global atmosphere over a large range of conditions. While the backscatter–to–extinction ambiguity has been appreciated for some time, there remains a legitimate need to develop methods to address this ambiguity, especially in the context of global spaceborne measurements. One approach around this problem is to develop methods capable of integrating additional information in the inversion process (e.g., the LIRAD method of *Platt* [1979]).

This paper introduces a method that can incorporate different information into the lidar retrieval problem as deemed relevant and is thus a potentially suitable candidate for application to spaceborne lidar data. The approach described in this paper inverts the lidar equation via an optimal estimation method popularized by the work of *Rodgers* [1976, 1990]. The lidar inversion problem is particularly well suited to this solution procedure given the simple analytical form of the forward model and its associated Jacobian (note equations (17c) and (19)). The retrieval method developed in this paper is illustrated using three examples drawn from LITE data; a case of thin cirrus observed along orbit 129 located at an altitude of about 16 km, a second case occurring along the same orbit corresponding to a (geometrically) thick aerosol layer near the surface, and a third case corresponding to the an aerosol layer observed along a portion of orbit 147 being lofted sufficiently far off the surface that the transmission through the layer could be determined. The retrievals obtained from this approach using only lidar backscatter as input were compared with retrievals performed using an iterative solution to the lidar inversion with the same input. The two methods produced essentially identical results.

The retrieval approach introduced in this paper offers a number of advantages over other methods, including (1) the method admits the incorporation of different kinds of information under a common retrieval philosophy, and this feature of the method is illustrated with the formal introduction of optical depth into the lidar inversion. It is argued that optical depth is a meaningful and desirable parameter to include in the lidar inversions for two reasons. Firstly, it is known (and further demonstrated in this study) that the optical depth derived from retrieved extinction profiles systematically varies with the value of the back-scattering-to-extinction ratio assumed for the retrieval. Secondly, optical depth information, in principle, could originate from a number of different measurement sources that may coincide with spaceborne lidar measurements. In this particular study, optical depths were derived directly from the lidar transmission estimated for the thin cirrus example and the lofted aerosol layer example and then applied in the retrieval. However, the method need not be restricted to this particular source of information. When optical depth is introduced, it becomes possible to retrieve the backscatter–to–extinction coefficient in addition to extinction profiles, given the caveat that the backscatter–to–extinction properties are uniform throughout the scattering layer. (2) The method provides a number of ways for evaluating the quality of the retrieval. Notably, the retrieval approach predicts full error diagnostics identifying sources of error due to measurement uncertainty (instrument noise and calibration uncertainty), model error (containing all the assumption built into the lidar equation and its parameters) as well as a priori error due to the influence of compiled databases on lidar backscatter of aerosol and cloud. When no optical depth information is available, the retrieval errors are large being dominated by the uncertainty attached to the backscatter-to-extinction ratio k. Under these circumstances the retrievals are only meaningful to the extent that k and its related uncertainty are known. When optical depth is introduced as a form of measurement, the error contributions shift to the extent that retrieval errors become dominated by the measurement error attached to the optical depth itself. The examples introduced in this paper were so constructed that the a priori error was negligible by virtue of the assumption that a priori information essentially was unreliable.

There are a number of issues that require further examination. For instance, the error analysis does not account for all possible sources of uncertainties, such as those associated with the assumption that k is constant through the scattering layer in question or with effects of multiple scattering. Furthermore, the optical depth constraint requires appropriate matching of optical depth to the lidar profile and this will be problematic to do in cases of multiple layers of scatterers. There are, however, also a number of ways the approach introduced in this paper could be extended to include even more information than considered in this study. Backscattering from a second lidar wavelength, more refined a priori data from other types of aerosol measurements, are among other possibilities that will be pursued in future research.

Appendix A: Addition of an Integral Constraint

An alternative way of introducing optical depth information into the lidar retrieval problem is to include this information as an additional constraint of integral form,

$$C(\hat{\mathbf{x}}) = \frac{\{\tau - G(\hat{\mathbf{x}})\}^2}{\sigma_\tau^2} \,, \tag{A1}$$

which is added to the cost function (6) where

$$G(\hat{\mathbf{x}}) = \Sigma_{\ell=2}^{N} \bar{x}_\ell \Delta R \,,$$

and σ_τ is the error assigned to the optical depth. As mentioned above, the source of this optical depth is immaterial provided both the values of τ and the associated error are specified. Adding this term to the cost function, we obtain the solution for its minimum [e.g., *Engelen and Stephens*, 1999]

$$\hat{\mathbf{x}} = \mathbf{x}_a + \mathbf{S}_a\mathbf{K}^T\mathbf{S}_y^{-1}[\mathbf{y} - f(\hat{\mathbf{x}})] + \Delta R\mathbf{S}_a\mathbf{L}^T\frac{\{\tau - G(\hat{\mathbf{x}})\}}{\sigma_\tau^2} , \tag{A2}$$

where $\mathbf{L}$ is a unit vector representing the derivative of $G(\hat{\mathbf{x}})$ with respect to $\hat{\mathbf{x}}$ divided by ΔR. This equation is then solved by iteration

$$\hat{\mathbf{x}}^{n+1} = \mathbf{S}_x^n([\mathbf{S}_a^{-1}\mathbf{x}_a + \mathbf{K}^{nT}\mathbf{S}_y^{-1}][\mathbf{y} - f(\hat{\mathbf{x}}^n) + \mathbf{K}^n(\hat{\mathbf{x}}^n)] + \tau\mathbf{L}\sigma_\tau^{-2}) . \tag{A3}$$

The error covariance matrix $\mathbf{S}_x$ is

$$\mathbf{S}_x^n = (\mathbf{S}_a^{-1} + \mathbf{K}^{nT}\mathbf{S}_y^{-1}\mathbf{K}^n + \Delta R\mathbf{L}^n\mathbf{L}^{nT}\sigma_\tau^2)^{-1} , \tag{A4}$$

and as before,

$$\mathbf{S}_x = \mathbf{D}_y\mathbf{S}_y\mathbf{D}_y^T + \mathbf{D}_a\mathbf{S}_a\mathbf{D}_a^T + \mathbf{D}_\tau\mathbf{S}_\tau\mathbf{D}_\tau^T , \tag{A5}$$

where

$$\mathbf{D}_\tau = \mathbf{S}_x\mathbf{K}\sigma_\tau^{-2} . \tag{A6}$$

To illustrate the method, cirrus retrievals were performed on the upper portion of the profile extracted from orbit 129. The optical depth of the thin cirrus layer was determined to be $\tau = 0.043$, and the uncertainty was estimated to be $\sigma_\tau = 0.1\tau$). The results shown in Figure A1 correspond to two different sets of retrievals derived assuming $k =$0.25 and 0.5, which approximately represents the spread of values of k expected for cirrus clouds. The two sets of results were created by systematically altering the S_y via a multiplicative factor and are presented as a function of the following parameter:

$$\psi = \frac{\Phi_y}{\Phi_\tau} , \tag{A7}$$

where

$$\Phi_y = (\mathbf{y} - f(\hat{\mathbf{x}}, \hat{\mathbf{b}}))^T\mathbf{S}_y^{-1}(\mathbf{y} - f(\hat{\mathbf{x}}, \hat{\mathbf{b}})) \tag{A8}$$

is the contribution to the cost function due to the difference between model and measurement and

$$\Phi_\tau = \frac{\{\tau - G(\hat{\mathbf{x}})\}^2}{\sigma_\tau^2} \tag{A9}$$

is the contribution to the cost function associated with the τ constraint. Values of $\psi < 1$ imply solutions weighted more by the forward model predictions, whereas $\psi > 1$ implies a solution correspondingly closer to that implied by the constraint. This is revealed in the figure showing how retrieval solutions systematically approach the constraint value $\tau = 0.043$ as ψ increases.

Figure A1.

There are two notable features of the solutions presented in Figure A1. The solution generally falls between the unconstrained solutions ($\psi = 0$) and a solution that produces an optical depth equivalent to that of the constraint. The solution approaches the latter as the model error increases. This error, however, cannot be made arbitrarily large without incurring the penalty of both large errors in retrievals as well as introducing retrieval solution instabilities, which result at the point highlighted for the $k = 0.25$ case. The second notable aspect of the solutions (and one not shown) is that the error is not ostensibly influenced by the constraint depite the more accurate nature of the latter and remains dominated by the model error largely due to the error attached to k.

Acknowledgments. This work was supported under NASA grant NASI-99103. TA acknowledges support from the Office of Naval Research, the National Aeronautic and Space Administration (GACP Progam and NAS1-99105), and the National Science Foundation ATM-0002198.

References

Ackerman, J., Extinction–to–backscatter ratio of tropospheric aerosol: A numerical study, *J. Atmos. Oceanic Technol.*, *15*, 1043–1050, 1998.

Alvarez, J., and M. Vaughan, Numerical calculation of cloud optical depth extinction from lidar, in *OSA Proceedings on the Inaugural Forum for the Research Center for Optical Physics*, vol. 19, pp. 90–95, 1994.

Anderson, T., S. Masonis, D. Covert, R. Charlson, and M. Rood, In situ measurements of the aerosol extinction–to–backscatter ratio at a polluted continental site, *J. Geophys. Res.*, *105*, 26,907, 2000.

Ansmann, A., M. Riesbesell, and C. Weitkamp, Measurement of atmospheric extinction profiles with raman lidar, *Opt. Lett.*, *15*, 746–748, 1990.

Bissonnette, L., et al., Lidar multiple scattering from clouds, *Appl. Phys. B*, *60*, 355–362, 1995.

Engelen, R. J., and G. L. Stephens, Characterization of water vapour retrievals from infrared TOVS radiances and microwave SSM/T-2 radiances, *Q. J. R. Meteorol. Soc.*, *125*, 331–351, 1999.

Fernald, F., Analysis of atmospheric lidar observations: Some comments, *Appl. Opt.*, *23*(5), 652–653, 1984.

Fernald, F., B. Herman, and J. Reagan, Determination of aerosol height distributions by lidar, *J. Appl. Meteorol.*, *11*, 482–489, 1972.

Ferrare, R., D. Turner, L. Brasseur, W. Feltz, O. Dubovik, and T. Tooman, Raman lidar measurements of the aerosol extinction-to-backscatter ratio over the southern great plains, *J. Geophys. Res.*, in press, 2001.

Grant, W., (Ed.), *Selected Papers on Laser Applications in Remote Sensing*, no. MS 141 in SPIE Milestone Series, SPIE Optical Engineering Press, 1997.

Grund, C., and E. Eloranta, The University of Wisconsin high spectral resolution lidar, *Opt. Eng.*, *30*, 6–12, 1991.

Jager, H., Lidar observed trend in stratospheric background aerosol, *Opt. Remote Sens. Atmos.*, *18*, 153–155, 1991.

Klett, J., Stable analytical inversion for processing lidar returns, *Appl. Opt.*, *20*, 211–220, 1981.

L'Ecuyer, T., and G. Stephens, An estimation-based precipitation retrieval algorithm for attenuating radars, *j. Appl. Meteorol.*, in press, 2001.

Marks, C., and C. Rodgers, A retrieval method for atmospheric composition from limb emission measurement, *J. Geophys. Res.*, *98*, 14,939–14,953, 1993.

Masonis, S., An empirical study of the lidar ratio and its variability, with implications for determining climate forcing by satellite-borne lidar, Ph.D. thesis, Univ. of Washington, Seattle, Wash., 2001.

Masonis, S., K. Franke, A. Ansmann, D. Mueller, D. Althausen, J. Ogren, A. Jefferson, and P. Sheridan, An intercomparison of aerosol light extinction and 180° backscatter as derived using in-situ instruments and Raman lidar during the INDOEX field campaign, *J. Geophys. Res.*, in press, 2001.

McCormick, M., P.-H. Wang, and L. Poole, Stratospheric aerosol and clouds, in *Aerosol-Cloud-Climate Interactions*, edited by P. Hobbs, Academic, San Diego, Calif., 1993.

Measures, R., *Laser Remote Sensing*, John Wiley, New York, 1984.

Menke, W., *Geophysical Data Analysis: Discrete Inverse Theory*, Academic, San Diego, Calif., 1989.

Miller, S., and G. Stephens, Multiple scattering effects in the lidar pulse stretching problem, *J. Geophys. Res.*, *104*, 22,205–22,219, 1999.

Muller, D., U. Wandinger, D. Althausen, I. Mattis, and A. Ansmann, Retrieval of physical properties from lidar observations of extinction and backscatter at multiple wavelengths, *Appl. Opt.*, *37*, 2260–2263, 1998.

Platt, C., Remote sensing of high clouds i: Calculation of visible and ir optical properties from lidar and radiometer measurements, *J. Appl. Meteorol.*, *18*, 1130–1143, 1979.

Platt, C., S. Young, R. Austin, G. Patterson, D. Mitchell, and S. Miller, LIRAD observations of tropical cirrus clouds in MC-TEX, part I, Optical properties and detection of small particles in cold cirrus, *J. Appl. Meteorol.*, in press, 2001.

Reagan, J., M. Apte, A. Ben-David, and B. Herman, Assessment of aerosol extinction to backscatter ratio measurements made at 694.3 nm in Tucson, Arizona, *Aerosol Sci. Technol.*, *8*, 215–226, 1988.

Rodgers, C., Retrieval of atmospheric temperature and composition from remote measurements of thermal radiation, *Rev. Geophys.*, *14*, 609–624, 1976.

Rodgers, C., Characterization and error analysis of profiles retrieved from remote sounding measurements, *J. Geophys. Res.*, *95*, 5587–5595, 1990.

Sassen, K., The polarization lidar technique for cloud research, *Bull. Am. Meteorol. Soc.*, *72*, 1848–1866, 1991.

Sassen, K., and J. Horel, Polarization lidar and synoptic analyses of an unusual volcanic aerosol cloud, *J. Atmos. Sci.*, *47*, 2881–2889, 1990.

Spinhirne, J., and W. Hart, Cirrus structure and radiative parameters from airborne lidar and spectral radiometer observations: The 28 October 1986 FIRE study, *Mon. Weather Rev.*, *118*, 2329–2343, 1990.

Spinhirne, J., J. Reagan, and B. Herman, Vertical distribution of aerosol extinction cross section and inference of aerosol imaginary index in the troposphere by lidar technique, *J. Appl. Meteorol.*, *19*, 426–438, 1980.

Stephens, G., *Remote Sensing of the Lower Atmosphere: An Introduction*, Oxford Univ. Press, New York, 1994.

Wandinger, U., A. Albert, J. Reichardt, and T. Deshler, Determination of stratospheric aerosol microphysical properties from independent extinction and backscattering measurements with Raman lidar, *Appl. Opt.*, *34*, 8315–8329, 1995.

Welton, E., K. Voss, P. Quinn, J. Campbell, J. Spinhirne, H. Gordon, and J. Johnson, Measurements of aerosol vertical profiles and optical properties during INDOEX 1999 using micro-pulse lidars, *J. Geophys. Res.*, in press, 2001.

Winker, D., and B. Wielicki, The PICASSO-CENA Mission, in *Sensors, Systems, and Next Generation Satellites*, edited by H. Fujisada, vol. 3870 of *Proc. SPIE*, p. 2636, 1999.

Winker, D., R. Couch, and M. McCormick, An overview of LITE: NASA's Lidar In-space Technology Experiment (LITE), *Proc. IEEE*, *84*, 164–180, 1996.

Wylie, D., P. Davis, W.Wolf, and E. Eloranta, Understanding satellite cirrus cloud climatologies with calibrated lidar optical depths, *J. Atmos. Sci.*, *52*, 4327–4343, 1995.

Young, S., Analysis of lidar backscatter profiles in optically thin clouds, *Appl. Opt.*, *34*, 7019–7031, 1995.

G. L. Stephens, Dept. of Atmospheric Science, Colorado State University, Fort Collins, CO 80523. (stephens@atmos.colostate.edu)

(Received March 16, 2001; revised July 9, 2001; accepted July 16, 2001.)

Paper number 2001JD000632.
0148-0227/01/2001JD000632$09.00

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

STEPHENS ET AL.: OPTIMAL ESTIMATION LIDAR RETRIEVALS

Figure Captions

Plate 1. Raw backscattering profiles along portions of orbits 129 (top) and 147 (bottom) of LITE. The location of the specific profiles used in the paper are indicated by the vertical dashed lines. The arrow indicates the location of the profiles used to create the retrievals presented in Figure 4.

Plate 1. Raw backscattering profiles along portions of orbits 129 (top) and 147 (bottom) of LITE. The location of the specific profiles used in the paper are indicated by the vertical dashed lines. The arrow indicates the location of the profiles used to create the retrievals presented in Figure 4.

Figure 1. Uncalibrated backscatter profiles averaged along approximately 25 km of orbit for the two locations highlighted in Plate 1. The optical depths noted are the values derived from lidar transmission analyses. The value quoted for the profile of orbit 129 corresponds to the cirrus layer only and does not include any contribution from the underlaying aerosol layer. The optical depth derived for the profile of orbit 147 corresponds to the aerosol layer located approximately between 1 and 6 km and does not include any contribution from the underlying thick scattering layer discernible below 1 km.

Figure 1. Uncalibrated backscatter profiles averaged along approximately 25 km of orbit for the two locations highlighted in Plate 1. The optical depths noted are the values derived from lidar transmission analyses. The value quoted for the profile of orbit 129 corresponds to the cirrus layer only and does not include any contribution from the underlaying aerosol layer. The optical depth derived for the profile of orbit 147 corresponds to the aerosol layer located approximately between 1 and 6 km and does not include any contribution from the underlying thick scattering layer discernible below 1 km.

Figure 2. Uncalibrated lidar return derived from that portion of orbit 147 identified in the top portion of Plate 1 at the location of the arrow. The raw backscatter profile is correlated to Rayleigh backscatter calculated from a given temperature profile. The slope of the dashed line (and the intercept) establishes relevant calibration factors applied to this specific LITE profile and used in the retrievals presented in Figure 6b.

Figure 2. Uncalibrated lidar return derived from that portion of orbit 147 identified in the top portion of Plate 1 at the location of the arrow. The raw backscatter profile is correlated to Rayleigh backscatter calculated from a given temperature profile. The slope of the dashed line (and the intercept) establishes relevant calibration factors applied to this specific LITE profile and used in the retrievals presented in Figure 6b.

Figure 3. Cirrus cloud backscatter–to–extinction ratio plotted as a function of midcloud temperature (modified from *Platt et al.* [2001]).

Figure 3. Cirrus cloud backscatter–to–extinction ratio plotted as a function of midcloud temperature (modified from *Platt et al.* [2001]).

Figure 4. Extinction profiles derived from the backscatter profile located along orbit 129 at the position indicated by the arrow in Plate 1. The retrievals shown correspond to the four values of k noted. The solid line represents the retrieval result obtained using the method introduced in this paper. The profile defined by the solid circles is the equivalent extinction profile derived from an alternative algorithm based on the iterative procedure of *Alvarez and Vaughan* [1994].

Figure 4. Extinction profiles derived from the backscatter profile located along orbit 129 at the position indicated by the arrow in Plate 1. The retrievals shown correspond to the four values of k noted. The solid line represents the retrieval result obtained using the method introduced in this paper. The profile defined by the solid circles is the equivalent extinction profile derived from an alternative algorithm based on the iterative procedure of *Alvarez and Vaughan* [1994].

Figure 5. Error characteristics corresponding to the (a) $k = 0.628$ and (b) $k = 0.157$ profiles of Figure 4. The respective extinction profiles and total error is shown in the left-most panel (the grey curve is the total extinction error) and the breakdown of this total error into a priori, measurement and model components.

Figure 5. Error characteristics corresponding to the (a) $k = 0.628$ and (b) $k = 0.157$ profiles of Figure 4. The respective extinction profiles and total error is shown in the left-most panel (the grey curve is the total extinction error) and the breakdown of this total error into a priori, measurement and model components.

Figure 6. (a) Cirrus cloud extinction profile retrieved for that portion of orbit 129 noted in Plate 1. As for Figure 5, the retrieved profiles derived from the two different methods are shown in the left panel. Also shown is the total error, and the retrieved value of k is quoted in the figure. The right-hand panel presents the errors as for Figure 5a and 5b but for the aerosol layer of orbit 147.

Figure 6. (a) Cirrus cloud extinction profile retrieved for that portion of orbit 129 noted in Plate 1. As for Figure 5, the retrieved profiles derived from the two different methods are shown in the left panel. Also shown is the total error, and the retrieved value of k is quoted in the figure. The right-hand panel presents the errors as for Figure 5a and 5b but for the aerosol layer of orbit 147.

Figure A1. Retrieval solutions for the cirrus case (refer Plate 1 and Figure 6a) expressed in terms of the retrieved optical depth plotted as a function of ψ which is a parameter that measures the relative strength of the imposed optical depth constraint (see text). The optical depth constraint is indicated by the horizontal line and is bordered by two dashed lines representing the 10 % range of uncertainty attached to this constraint. Two retrievals are shown, one for an assumed $k = 0.25$ and another for $k = 0.5$. The point at which the retrieval for $k = 0.25$ fails is indicated by the star symbol.

Figure A1. Retrieval solutions for the cirrus case (refer Plate 1 and Figure 6a) expressed in terms of the retrieved optical depth plotted as a function of ψ which is a parameter that measures the relative strength of the imposed optical depth constraint (see text). The optical depth constraint is indicated by the horizontal line and is bordered by two dashed lines representing the 10 % range of uncertainty attached to this constraint. Two retrievals are shown, one for an assumed $k = 0.25$ and another for $k = 0.5$. The point at which the retrieval for $k = 0.25$ fails is indicated by the star symbol.

Tables

Table 1. Lidar Ratio Measurements With Significant Sample Sizes

Method	Location	N[a]	Sa range (sr)[b]	Notes[c]
Slant lidar	Tucson	57	15 - 82	1
Nephelometry	central Illinois	95	27 - 75	2
Nephelometry	Indian Ocean	168	33 - 91	3
Nephelometry	Hawaiian coast	35	26 - 33	4
Lidar/sunphotometer	Indian Ocean	102	25 - 75	5
Raman lidar	central Oklahoma	2556	45 - 87	6

[a] N is the estimated number of independent samples.

[b] Sa range is the range that encompasses 95% of the data.

[c] 1, Lidar measurements at several slant angles determines effective S over mixed layer at 694 nm; sporadic daily samples over 5 years; range and N reflect only samples with less than 30% uncertainty [*Spinhirne et al.*, 1980; *Reagan et al.*, 1988]; 2, nephelometric method determines local S at 532 nm; continuous surface measurements over 5 weeks; N assumes one independent sample every 4 hours [*Anderson et al.*, 2000]; 3, method as in note (2); airborne data (30 to 3500 m altitude) from 18 flights over 6 weeks during low-level offshore flow from surrounding continents; N assumes one independent sample every 1 km vertical or 100 km horizontal [*Masonis*, 2001; *Masonis et al.*, 2001]; 4, method as in note (2); continuous surface measurements over 8 days during onshore (marine) flow; N assumes one independent sample every 4 hours [*Masonis*, 2001]; 5, vertically pointing micropulse lidar and sunphotometer determine effective S over aerosol-laden lower troposphere; continuous data from ship during same experiment as in note (3); N assumes one independent sample every 4 hours for the 17 cloud-free days of data [*Welton et al.*, 2001]; 6, Raman lidar determines local S from 0.8 km to about 4 km at 355 nm; continuous data from surface station over 2 years; N assumes four independent samples in vertical obtained every 4 hours over 213 days with 50% data loss due to cloud contamination [*Ferrare et al.*, 2001].

Table 2. Aerosol Layer Backscatter-to-Extinction Ratios and Retrieval Optical Depths Associated With Results of Figure 4.

S (sr)	K	τ
20	0.620	0.060 (0.061)
40	0.314	0.147 (0.149)
60	0.209	0.285 (0.289)
80	0.157	0.525 (0.526)