Lecture on Energy Balance Modeling by Cecilia Bitz

Case 0: Zero-dimensional climate model

The simplest model is a balance of incoming and outgoing radiant energy

$$< N > = < S > - < F >$$

where N	=	Net radiative flux
S	=	absorbed solar by atmosphere and surface (planet)
F	=	outgoing planetary longwave
$<>$	=	global average and annual mean average

$$< S > = \frac{S_\odot}{4}(1 - < \alpha_p >)$$

where $S_\odot$= solar constant 1370 Wm^{-2} and α_p = planetary albedo

The 4 comes about from the ratio of the Earth's surface $4\pi r^2$ to the area of Earth's disk that intercepts sunlight πr^2

Solar constant

The sun emits radiation approximately like a blackbody at about $T_\odot = 6000$ Kelvin. If a body is "black", its total emission is determined only by its temperature. Hence total radiant energy $= \sigma T_\odot^4$ (Stefan-Boltzmann Law), where $\sigma = 5.67\ 10^{-8}$ Wm^{-2} K^{-4} (remember by 5,6,7,8).

half solar energy is VIS and NUV (0.2-0.7 microns) and half is NIR (0.7-5 microns).

$S_\odot$is easy to compute from radius of sun $r_\odot$ and earth-sun distance r_{es}

$$S_\odot = \sigma T_\odot^4 \frac{4\pi r_\odot^2}{4\pi r_{es}^2}$$

It is measured from space to be 1370 Wm^{-2}. A black solar collector of area 1 m^2 situated just above Earth's atmosphere and oriented perpendicular to the solar beam will collect 1370 W

Keep in mind that the solar constant is not very constant.

- It is different for each planet
- Earth's orbit is eccentric $-$ r_{es} varies by +/-1.75% over course of year so $S_\odot$varies by +/-3.5%
- Varies as sun rotates with 29-day cycle bringing sun spots groups into view
- Varies with the 11-year magnetic cycle which results in sunspot cycle max at 1979, 1990, 2001...

- Varies with sunspot activity, possible explanation for Little Ice Age and increase in global temperature during the 1920s both coincident with time of sunspots extrema.
- Increases with age, roughly 10% per billion years

Planetary Albedo - α_p

Definition: Fraction of insolation (TOA incoming solar radiation) reflected back to space mostly by cloud but also by surface, aerosols, and gases. Not the same as surface albedo.

insolation is distributed by

30%	reflection to space
25%	absorbed by atmosphere (clouds, aerosols, & gases)
45%	absorbed by surface

Thus $< \alpha_p >= 0.3$

Over most surfaces, $\alpha_s = 0.1$ to 0.2 so $\alpha_p > \alpha_s$, except over snow where $\alpha_s \sim 0.8$ and $\alpha_p \sim 0.7$

$$\text{Finally} \quad < S >= 0.7 \frac{1370 \mathrm{Wm}^{-2}}{4} = 240 \mathrm{Wm}^{-2}$$

Outgoing planetary longwave

Measurements from space of outgoing planetary longwave (also called thermal or infrared) radiation from Earth show $< F >= 240 \mathrm{Wm}^{-2}$. Because the geothermal heat flux is insignificant at 0.05 Wm^{-2}, Earth is in energy balance.

$< F >$ is primarily in the range 4-50 microns. Like the sun, the Earth is also, to a good approximation, a blackbody (especially the surface, including snow and ice, and clouds). Hence

$$< F >= \sigma < T >^4$$

A flux of 240 Wm^{-2} implies a blackbody temperature of 255 K (-18 C). This is about the temperature of the 6-km height in the atmosphere. Thus the thermal photons emerging at the TOA are on average emitted from 6 km.

Gases are not good blackbody radiators. They absorb and emit radiation in selective bands.

The surface, which absorbs nearly half of the insolation, is a heat source for the lower atmosphere (troposphere). The temperature on average decreases with vertical temperature lapse rate -6.5 K/km, which is affected by radiative heating and vertical convection.

Earth's atmosphere absorbs most of terrestrial (surface) longwave. The absorbers re-emit radiation both up and down. If the earth had no atmosphere and α_p was still 0.3, the surface would have to be 255 K (-18 C) in equilibrium. Present day the global mean surface temperature is about 15 C. The 33 °difference is due to the "greenhouse effect". The globally averaged upward surface longwave is 390 Wm^{-2}, but only 240 escapes at TOA, so the global mean greenhouse effect is 150 Wm^{-2} due to the insulating effect of the atmosphere.

Since 1900 humans have added enough CO2 CH4 N20 and CFC to trap an additional 3 Wm^{-2}. How this additional heating translates into temperature change is complicated by feedbacks.

Climate Sensitivity

Return to zero-dimensional climate model

$$<N>=<S>-<F>$$

Recall $<N>=0$ in equilibrium, but under climate perturbation

$$\Delta<N>=\Delta<S>-\Delta<F>$$

where Δ represents perturbation to the quantity. For a balance to be re-established, it is assumed that the surface temperature will change by ΔT_s

$$\Delta<T_s>\left[\frac{\Delta<F>}{\Delta<T_s>}-\frac{\Delta<S>}{\Delta<T_s>}\right]=-\Delta<N>=G$$

where G is the direct radiative forcing to the climate system (e.g., 2xCO2 yields G=4 Wm^{-2})

The relation between T_s and G is

$$\Delta<T_s>=\lambda G,$$

where

$$\lambda=\frac{1}{\frac{\Delta<F>}{\Delta<T_s>}-\frac{\Delta<S>}{\Delta<T_s>}}$$

is the "climate sensitivity parameter". λ determines the response of the climate system. Taylor's series expansions yield

$$\frac{\Delta<F>}{\Delta<T_s>}=\frac{\partial<F>}{\partial<T_s>}+\frac{\partial<F>}{\partial<W>}\frac{\Delta<W>}{\Delta<T_s>}+\frac{\partial<F>}{\partial<C>}\frac{\Delta<C>}{\Delta<T_s>}+...$$

and

$$\frac{\Delta<S>}{\Delta<T_s>}=\frac{S_\odot}{4}\left[\frac{\partial<\alpha_p>}{\partial<T_s>}+\frac{\partial<\alpha_p>}{\partial<C>}\frac{\Delta<C>}{\Delta<T_s>}+\frac{\partial<\alpha_p>}{\partial<V>}\frac{\Delta<V>}{\Delta<T_s>}+...\right]$$

where W is column averaged water vapor amount, C is cloud amount, and V is vegetative cover.

Consider a minimal temperature dependence: $< F >= \sigma < T_s >^4$ with $< S >$ independent of temperature

$$\lambda = \frac{1}{\frac{\partial < F >}{\partial < T_s >}} = \frac{< T_s >}{4 < F >} = 0.3 \text{ K W}^{-1} \text{ m}^2.$$

This can be considered λ for a "reference" climate because the temperature dependence appears only in $< F >$. Keep in mind one person's feedback is another person's reference system, forcing, or something else. The definition of feedback is not consistent in the climate literature.

However we already learned that for Earth, this is a poor approximation for $< F >$ because of the greenhouse effect. Yet it is the surface temperature, rather than the planetary temperature or the 6-km height temperature, that is more likely to be useful as a criterion for information about the cryosphere.

A common relation is $F = A + BT_s$. This is an example of a parameterization, a simple equation representing the net effect of many processes. A linearization of σT_s^4 would give a slope of B much larger than is obtained from observations, because of the water-vapor positive feedback. A=203.3 Wm^{-2} and B=2.09 $\text{W m}^{-2}\ {}^\circ\text{C}^{-1}$ are determined from satellite measurements.

I will put off a formal mathematical definition of feedback temporarily. In general feedbacks cause a higher (for positive feedback) or lower (for negative feedback) temperature change than the reference system experiences alone. For example, an increase in relative humidity caused by an increase in T_s, makes the atmosphere more opaque to longwave so that it radiates to space from a higher height than the reference climate system. Ultimately the surface temperature increases more than the reference system for this example of a positive feedback.

Another well known feedback is the temperature-albedo feedback. Imbrie and Imbrie book has wonderful history of the 19th century character James Croll, who is credited with the first description of the temperature-albedo feedback. In this case α is a function of T_s.

Case 1: One-dimension Model

Add latitude to zero-dimensional model and still consider only the annual mean, steady state problem.

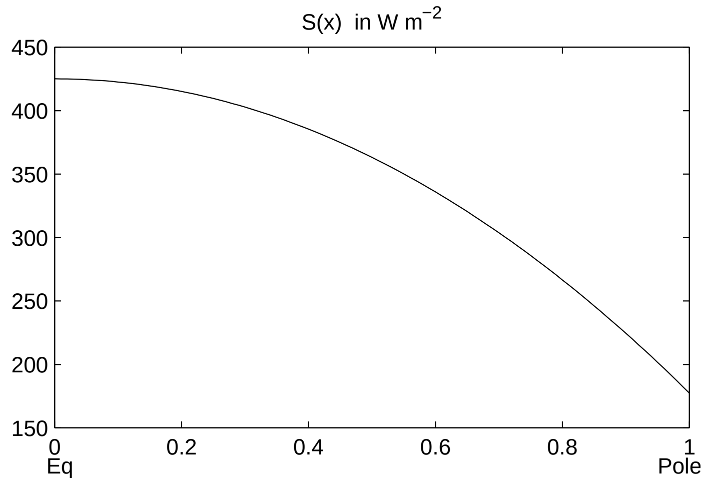
$$0 = S(\phi)(1 - \alpha(\phi)) - F(\phi)$$

Let $x = sin(\phi)$ so even spacing in x results in even spacing in horizontal area along latitude circles (i.e., $dx = cos(\phi)d\phi$).

The latitudinal distribution of the annual mean insolation is $S(x) = Qf(x)$ where $Q = S_\odot/4$ and the latitudinal variations are described by $f(x)$, which can be approximated by a series expansion in even order Legendre Polynomials:

$$f(x) \sim 1 - 0.482 \left(\frac{3x^2 - 1}{2} \right).$$

In this case taking just the first two even polynomials is accurate to about $+/-2\%$.



The planetary albedo is

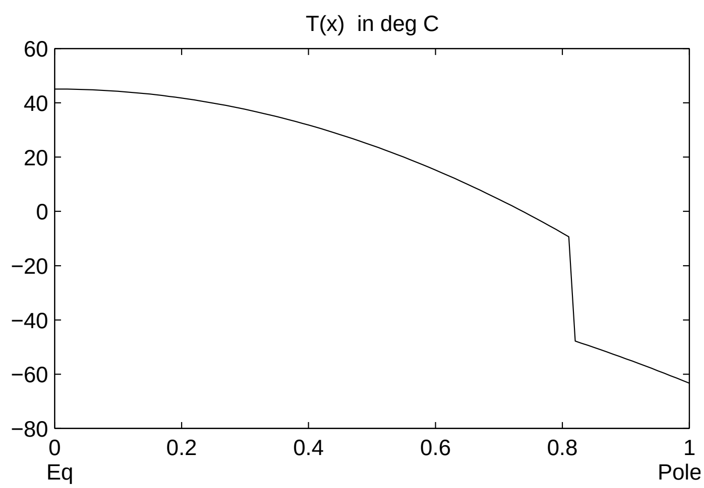
$$\alpha(x) = \begin{cases} 0.6; & \text{ice or } T(x) < -10°C \\ 0.3; & \text{no ice or } T(x) > -10°C. \end{cases}$$

The outgoing longwave radiation is

$$F(x) = A + BT(x).$$

The solution to this simple model is

$$T(x) = \frac{Qf(x)(1-\alpha(x)) - A}{B}$$



The ice line x_s is defined as the minimum latitude that is ice covered which in this case is $x_s = .8$ or $\phi = 53°$.

The pole to equator temperature difference in this case is about 110°C while the observed value is only 40°C. Past climates had about 20°C during the Eocene, about 50 million years ago, and about 50°C during the last glacial maximum, about 25 thousand years ago.

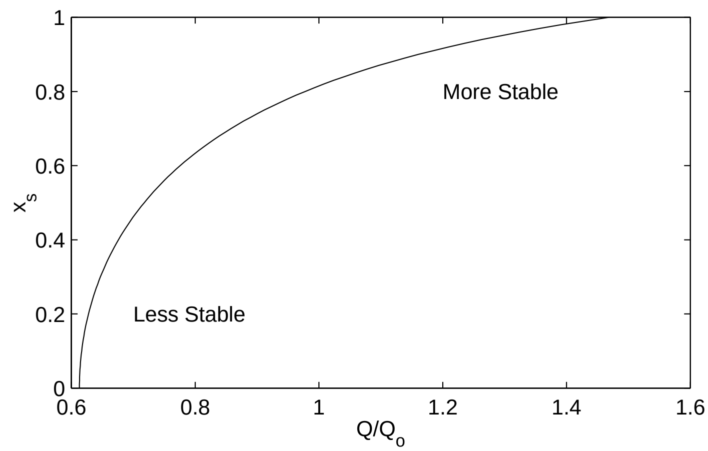
The climate stability in the context of energy balance model is often based on the stability of the position of the ice line to perturbations in Q or more precisely:

$$\sim \left(\frac{\partial x_s}{\partial Q}\right)^{-1}.$$

It is easier to solve for Q for a given x_s

$$Q = \frac{A + BT(x_s)}{(1 - \alpha(x_s))f(x_s)}.$$

Using $T_s = -10°C$, $\alpha_s = 0.3$, and Q_o to denote the present day mean insolation of the planet gives the stability curve in the following figure. The slope of the curve is a measure of the stability. In this case the curve is everywhere positive and hence the model is stable for any Q.



Case 2: One-dimension Model with Atmospheric Heat Transport

Let R denote atmospheric heat transport. R is constrained to be zero at the poles, due to converging meridians, and the integral of the heat transport across all latitudes must be zero. In this case, R is approximated by a diffusive process (after North 1975) and hence

$$R = -D\frac{dT}{d\phi} \qquad \frac{d}{d\phi} = cos(\phi)\frac{d}{dx}$$

$$R = -D(1 - x^2)^{1/2}\frac{dT}{dx}$$

However, the heat budget depends on the convergence of the heat transport which is

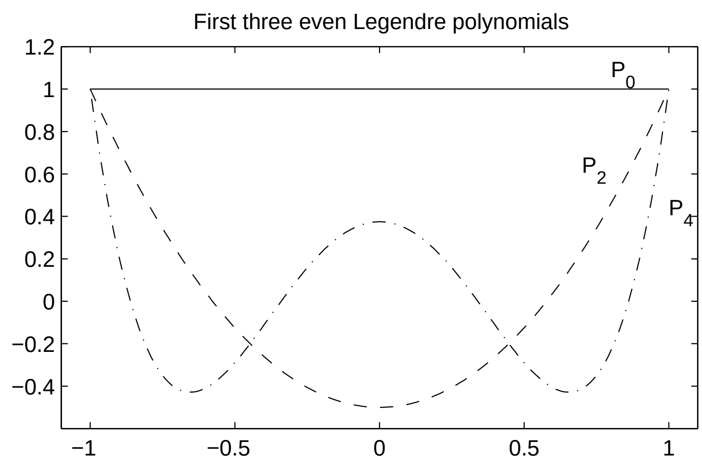
$$\frac{d}{dx}D(1 - x^2)\frac{dT}{dx}.$$

A simple way to think of this is that the heat deposited in a latitude band of width $d\phi$ is the heat transport entering the latitude band say from the south minus the heat transport exiting the band in the north.

Finally the equation for the model with atmospheric heat transport is

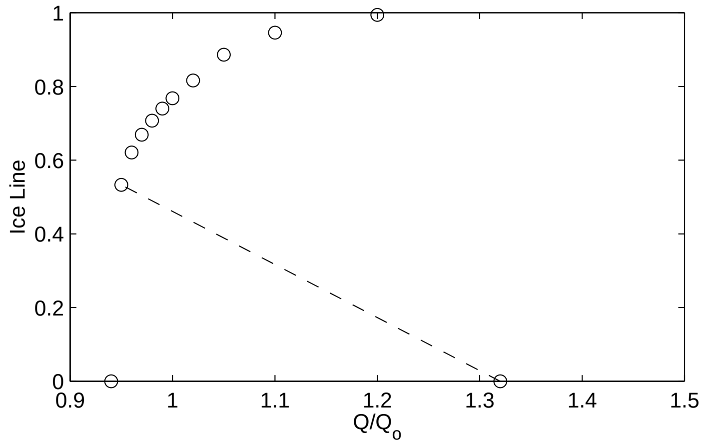
$$Qf(x)(1-\alpha(x)) = A + BT - \frac{d}{dx}D(1-x^2)\frac{dT}{dx}.$$

This equation can be solved exactly (See North 1975) where the temperature is a series expansion of Legendre Polynomials. If $f(x)$ is approximated as just the sum of the first two even Legendre polynomials (only the even order polynomials are needed to describe the annual mean distribution of insolation upto $+/-2\%$), then $T(x)$ is also just the sum of the first two Legendre polynomials with coefficients determined by solving the equation above. The first three even Legendre polynomials (P_0, P_2, and P_4) are shown in the figure below. In this case the coefficient of P_0 determines the global mean temperature and the coefficient of P_2 determines the pole to equator temperature difference.



One clear problem with this model is that the temperature gradient with latitude is too great in the tropics. This can be easily improved by increasing D in the tropics. Once this modification is made to the model, it can no longer be solved analytically and a model must be used. At this point, $D(x)$ can be tuned to achieve a good match to the data. This is an example of model tuning. It is important to be aware that $D(x)$ cannot be tuned independent of all the other model variables.

The addition of heat transport to our simple energy balance model fundamentally changes the stability of model's climate as can be seen in the figure below. The stability curve was constructed from model simulations by varying Q/Q_o and the initial conditions in a series of simulations.


Ice Line
Q/Q_o
1
0.8
0.6
0.4
0.2
0
0.9
1
1.1
1.2
1.3
1.4
1.5