Lecture 8. Parameterization of BL Turbulence I

In this lecture…

- Fundamental challenges and grid resolution constraints for BL parameterization
- Mixed layer models for convective BLs
- Turbulence closure (e. g. first-order closure and TKE) parameterizations
- *K*-profile and nonlocal parameterizations

Turbulence parameterization and the turbulence closure problem (G2.4-5)

The goal of a turbulence parameterization is to predict tendencies (time derivatives) of all prognostic variables (fluid velocity components, temperature, moisture and other advected constituents or 'tracers') at all gridpoints of a numerical model due to unresolved turbulent motions. We will restrict ourselves to large-scale forecast and climate models in which the horizontal grid spacing is insufficient to resolve the most energetic BL turbulent eddies, which might be tens of meters to 1-2 km across, so we assume *all turbulence is unresolved*. We will assume that ensemble-averaged horizontal gradients of advected quantities are much less than their vertical gradients in the BL, so that *horizontal turbulent flux convergence can be neglected compared to vertical turbulent flux convergence*. In a given grid column, we let $\bar{a}(z,t)$ be the turbulent-ensemble mean profile of some advected constituent *a*, and *a'* be an unresolved turbulent fluctuation about $\bar{a}$. For simplicity, we use the Boussinesq approximation.

With these assumptions,

$$\left(\frac{\partial \bar{a}}{\partial t}\right)_{\text{turb}} = -\frac{\partial}{\partial z}\left(\overline{w'a'}\right) \tag{8.1}$$

That is, to predict the horizontal mean $\bar{a}$ (a first-order moment of the horizontal pdf of *a* across the grid cell) we need to specify the vertical profile of the turbulent flux $\overline{w'a'}$ (a second-order joint moment of *w* and *a* of the horizontal turbulent pdf). There is no rigorously correct way to find the flux in terms of the horizontal mean values of *a* and other variables. Fundamentally, this issue arises because of the advective nonlinearity in the fluid transport equations.

There are two ways to proceed. The first is introduce some assumption or '**first-order closure**' which represents this flux in terms of the mean variables; this is simple and popular, though it has well-known inaccuracies for convective boundary layers.

The second is to derive an equation for the flux from the Boussinesq equations. For the simplest case of no horizontal mean flow, quoting (G Eq. 2.64a),

$$\frac{\partial}{\partial t}\left(\overline{w'a'}\right) = -\overline{w'w'}\frac{\partial \bar{a}}{\partial z} - \frac{\partial}{\partial z}\overline{w'w'a'} + \overline{a'b'} - \frac{1}{\rho}\overline{a'\frac{\partial p'}{\partial z}} \tag{8.2}$$

where *b* and *p* are buoyancy and pressure. This equation involves a *third* moment $\overline{w'w'a'}$, which must somehow be represented, as well as other second-order moments. We could make assumptions (called second-order closure) relating the third moments to second moments, or we could find prognostic equations for the third moments – but these will involve fourth moments. In the end, there is no escaping making some '**turbulence closure**' assumption to specify unknown turbulent moments in terms of known ones. Generally, using a suitable higher-order turbulence closure will be more accurate than using a lower-order closure, but also involves many more prognostic equations that often require a short integration timestep. Some large-scale models do have turbulence parameterizations based on second and third-order turbulence closure and such closures have attractive features for cloud-topped boundary layer turbulence, but their mathematical complexity and discretization issues have limited their popularity to date.

Large-eddy simulations also generally include parameterizations for unresolved subgridscale (SGS) turbulent fluxes. The basic concepts and approaches used in LES SGS schemes are similar to those used in large-scale models. However, hese parameterizations differ from those in large-scale models in several ways. First, the LES is simulating the larger turbulent eddies, so the goal is to simulate the effect of subgrid turbulent eddies on the resolved-scale turbulent motions, rather than to simulate the entire turbulent flux. Second, the large turbulent eddies resolved in an LES have comparable horizontal and vertical scales, so horizontal subgrid turbulent fluxes must be predicted as well as vertical fluxes. Third, because LES SGS parameterizations must operate at every gridpoint and on the short LES timestep, they must be computationally simple.

Vertical resolution and discretization considerations for BL parameterizations

Fig. 8.1 shows the distribution of thermodynamic grid levels in the lowest 20% of the atmosphere for three state-of-the-art models- the NCAR Community Atmosphere Model version 5 (CAM5, 30 levels overall) used for climate modeling, the ECMWF global operational forecast model (91 levels overall), and the WRF mesoscale model as used at the University of Washington for real-time short-range weather forecasting in the Pacific Northwest (37 levels overall). For climate models, the time step can be up to an hour, requiring careful attention to the numerical stability of BL parameterizations and their interactions with other parts of the model.

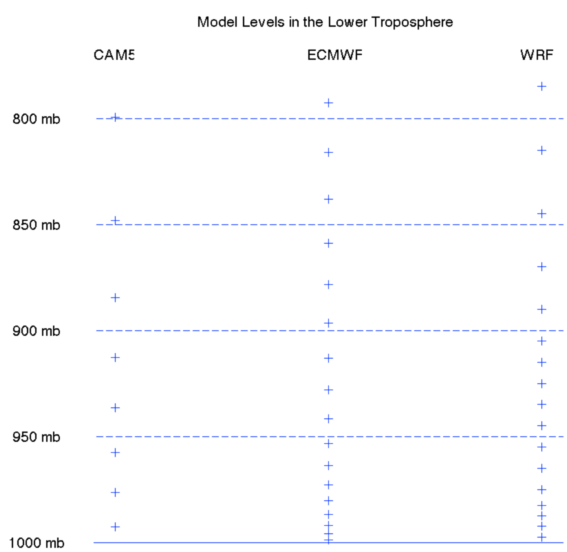

Fig. 8.1: Grid levels for three current atmospheric models, assuming the surface is at 1000 hPa.

It is highly desirable to use advected variables that mix approximately linearly and are conserved in adiabatic vertical displacement (such as theta or water vapor mixing ratio in cloud-free air; we will reserve discussion of appropriate conserved variables for cloudy boundary layers for later). The turbulent tendency of such variables can be approximated as the corresponding vertical turbulent flux convergence.

BL parameterization types

In the next two lectures, we discuss three BL parameterization approaches. One easy to understand parameterization approach, **mixed-layer modeling**, is often used in idealized models of convective BLs (and for the near-surface layer of the ocean). The two parameterization approaches that are most popular for use in forecast models are **'local' eddy diffusivity** parameterizations derived from turbulence closure assumptions) and **non-local K-profile** parameterizations. These approaches are capable of representing all boundary layer types.

Mixed Layer Models

Mixed layer models (MLMs) assume that the mean vertical profiles of u, v, and conserved tracers such as θ are uniform ('**well-mixed**') across the BL, and the BL is capped by a sharp inversion at which temperature and moisture have sudden jumps. MLMs are most applicable to convective BLs and poorly represent most stable BLs. MLMs are mainly used by researchers and teachers as a conceptual tool for understanding the impacts of different physical processes on BL turbulence. One GCM (run at UCLA) uses a mixed layer model to describe the properties and the variable depth of its lowest grid layer all over the globe. The key turbulence closure in a MLM is the **entrainment closure**, an empirically-inspired relationship between the rate that mass is entrained from above the BL and known variables of the MLM such as surface fluxes and inversion jumps.

For simplicity, we will consider a case with no horizontal advection or mean vertical motion, no thermal wind, and no diabatic effects above the surface. We will assume that the surface momentum and buoyancy fluxes are given (in general, these will depend upon the mixed layer variables, but we needn't explicitly worry about this now). We let $h(t)$ be the mixed layer top, at which there may be jumps in the winds and potential temperature, denoted by Δ. Turbulence in the mixed layer entrains free-tropospheric air from just above the mixed layer, causing h to rise at the **entrainment rate** w_e.

$$\frac{du_M}{dt} - f(v_M - v_g) = -\frac{\partial}{\partial z}\overline{u'w'}, \quad (8.3)$$

$$\frac{dv_M}{dt} + f(u_M - u_g) = -\frac{\partial}{\partial z}\overline{v'w'}, \quad (8.4)$$

$$\frac{d\theta_M}{dt} = -\frac{\partial}{\partial z}\overline{w'\theta'}, \quad (8.5)$$

$$\frac{dh}{dt} = w_e . \quad (8.6)$$

Since the left hand sides of (8.3-8.5) are height-independent, the right hand sides must be, too, so the turbulent fluxes of u, v, and q are linear with z in the mixed layer (note that this would no longer be the case for a baroclinic BL in which u_g varied with height, or in the presence of internal sources or sinks of θ) . This allows the turbulent fluxes to be written in terms of the surface and BL top fluxes:

$$\overline{w'a'}(z) = (1-\zeta)\overline{w'a'}(0) + \zeta\overline{w'a'}(h), \quad \zeta = z/h. \quad (a = u,\ v, \text{ or } \theta). \tag{8.7}$$

The turbulent fluxes are prescribed at the surface and can be deduced at the mixed-layer top h as follows. The entrainment deepening of the BL, in which free-tropospheric air with value a^+ of is replaced by BL air with $a = a_M$ at the rate w_e, requires an upward turbulent **entrainment flux** into the mixed layer top of

$$\overline{w'a'}(h) = -w_e\Delta a, \quad \Delta a = a^+ - a_M. \tag{8.8}$$

Thus the flux convergence terms can be written:

$$-\frac{\partial}{\partial z}\overline{w'a'} = \frac{-w_e\Delta a - \overline{w'a'}(0)}{h}. \tag{8.9}$$

This closes the set of equations (8.3-8.6) except for a specification of w_e, called the **entrainment closure**. This is the big assumption in any MLM and is typically based . For cloud-free unstable to nearly neutral mixed layers, formulas such as that inferred in Lecture 7 (Moeng and Sullivan 1994) have solid observational and LES grounding and are commonly used:

$$\overline{w'b'}(h) = -w_e\Delta b = -(0.2w_*^3 + u_*^3)/h, \quad \Delta b = g\Delta\theta_v/\theta_0 \text{ (inversion \textbf{buoyancy jump})}. \tag{8.10}$$

Recall that w_* and u_* are determined by the surface buoyancy and momentum fluxes, respectively, so this closure determines w_e in terms of known variables. In the purely convective BL with no mean wind, $u_*^3 = 0$, $w_*^3 = B_0h$, so the above relation reduces to

$$\overline{w'b'}(h) = -0.2B_0 \tag{8.11}$$

A classic application of a MLM (presented in Lecture 12) is to the deepening of a convective boundary layer due to surface heating.

Local (Eddy diffusivity) parameterizations (Garratt 8.7)

In eddy-diffusivity (often called ***K*-theory**) models, the turbulent flux of an adiabatically conserved quantity a (such as q in the absence of saturation, but not temperature T, which decreases when an air parcel is adiabatically lifted) is related to its gradient:

$$\overline{w'a'} = -K_a\frac{d\overline{a}}{dz} \tag{8.12}$$

The key question is how to specify K_a in terms of known quantities. Three approaches are commonly used in mesoscale and global models:

i) **First-order closure**, in which K_a is specified from the vertical shear and static stability.

ii) **1.5-order closure** or **TKE closure**, in which TKE is predicted with a prognostic energy equation, and K_a is specified using the TKE and some lengthscale.

iii) ***K*-profiles**, in which a specified profile of K_a is applied over a diagnosed turbulent layer depth based on surface fluxes or other vertically-aggregated forcings for turbulence.

From here on we will drop overbars except on fluxes, so $a(z)$ will refer to an ensemble or horizontal average at level z. The following discussion of these approaches is necessarily oversimplified; a lot of work was done in the 1970's on optimal ways to use them. An excellent review of first, 1.5, and second-order closure is in Mellor and Yamada (1982, *Rev. Geophys. Space Phys.*, **20**, 851-875).

First-order closure

We postulate that K_a depends on the vertical shear $s = |du/dz|$, the buoyancy frequency N^2, and an eddy mixing lengthscale l. In most models, saturation or cloud fraction is accounted for in the computation of N^2. A Richardson number $\mathrm{Ri} = N^2/s^2$ is defined from the shear and stability. Dimensionally,

$$K_a = \text{length}^2/\text{time} = l^2 s F_a(\mathrm{Ri}) \tag{8.13}$$

One could take the stability dependence in $F_a(\mathrm{Ri})$ the same as found for the surface layer in Monin-Obukhov theory, e. g. $F_a(\mathrm{Ri}) = [\phi_a(\zeta)\,\phi_m(\zeta)]^{-1}$, where z depends on Ri as in the surface layer, and $\phi_a = \phi_m$ (if a is momentum) or ϕ_h (if a is a scalar). This works reasonably in a weakly stable BL. In the convective BL it gives (see page 6.2) $F_m(\mathrm{Ri}) = (1 - 16\mathrm{Ri})^{1/2}$ and $F_h(\mathrm{Ri}) = (1 - 16\mathrm{Ri})^{3/4}$. However, in nearly unsheared convective flows, one would like to obtain a finite K_a independent of s in the limit of small s. This requires $F_a \propto (-\mathrm{Ri})^{1/2}$ so $K_a \propto l^2 s(-\mathrm{Ri})^{1/2} = l^2(-N^2)^{1/2}$. This is consistent with the M-O form for K_m but not for K_h. Thus, we just choose $K_m = K_h$ to obtain:

$$F_{h,m}(\mathrm{Ri}) = \begin{cases} (1-16\mathrm{Ri})^{1/2} & (\text{unstable}, \mathrm{Ri} < 0) \\ (1-5\mathrm{Ri})^2 & (\text{stable}, 0 < \mathrm{Ri} < 0.2) \end{cases} \tag{8.14}$$

No turbulent mixing is diagnosed if Ri > 0.2. Every model has its own form of $F(\mathrm{Ri})$, but most are qualitatively similar to this. Usually, if this form is used within the stable BL, the F's are enhanced near the surface (e. g. no Ri = 0.2 cutoff) to account for unresolved flows and waves driven, for instance, by land-sea or hill-valley circulations that can result in spatially and temporally intermittent turbulent mixing.

Many prescriptions for l exist. The only definite constraint is that $l \to kz$ near the surface to match (8.13) to the eddy diffusivity in neutral conditions to that observed in a log layer, $K_a = ku_*z$. One commonly used form for l (suggested by Blackadar, 1962) is

$$l = \frac{\lambda}{1 + \lambda / kz} \tag{8.15}$$

where the `asymptotic lengthscale' λ is chosen by the user. A typical choice is λ = 50-100 m, or roughly 10% of the boundary layer depth. The exact form of λ is less important than it might appear, since typically there will be (i) layers with large K_a, and small gradients (i. e. fairly well mixed layers) in which those small gradients will just double (but still be small) if K_a, is halved, to maintain the same fluxes, (ii) layers with small K_a where physical processes other than turbulence will tend to dictate the vertical profiles of velocity and temperature, and (iii) a surface layer, in which the form of K_a is always chosen to match Monin-Obukhov theory, and so is on solid observational ground.

1.5-order closure

In 1.5-order (or TKE) closure, we prognose the TKE $e = q^2/2$ based on the shear and stability profiles and use the TKE in computing the local eddy diffusivity. This allows representation of TKE storage and transport, which are important in convective boundary layers. It is called 1.5-order closure because it lies between first-order and second-order turbulence closure, since we prognose one second-order moment (TKE) but diagnose all other second-order moments.

Using the same eddy mixing lengthscale as above, dimensionally

$$K_a = (\text{length})(\text{velocity}) = lqS_a(G_m, G_h), \qquad a = m \text{ (momentum) or } h \text{ (heat)} \quad (8.16)$$

$$G_m = l^2S^2/q^2, \quad G_h = -l^2N^2/q^2 \quad (8.17)$$

Closure assumptions and measurements discussed in Mellor and Yamada dictated the form of S_M and S_H in terms of the nondimensional shear and stratification G_m and G_h, involving the numerical solution of a set of simultaneous nonlinear algebraic equations. Galperin (1988) argued that Mellor and Yamada's closure assumptions were consistent with a further simplification of their equation set, giving the following forms plotted in Fig. 8.2:

$$S_M = \frac{\alpha_1 + \alpha_2 G_H}{(1+\alpha_3 G_H)(1+\alpha_4 G_H)}, \quad S_H = \frac{\alpha_5}{1+\alpha_3 G_H}, \ -0.28 < G_H < 0.0233$$

$$\alpha_1 = 0.5562,\ \alpha_2 = -4.364,\ \alpha_3 = -34.6764,\ \alpha_4 = -6.1272,\ \alpha_5 = 0.6986$$

As in first order closure, the stability functions are larger in unstable stratification ($G_h > 0$) than in stable stratification ($G_h < 0$). This formulation is used in the UW boundary layer scheme implemented in CAM5 and WRF (Bretherton et al. 2004; Bretherton and Park 2009).

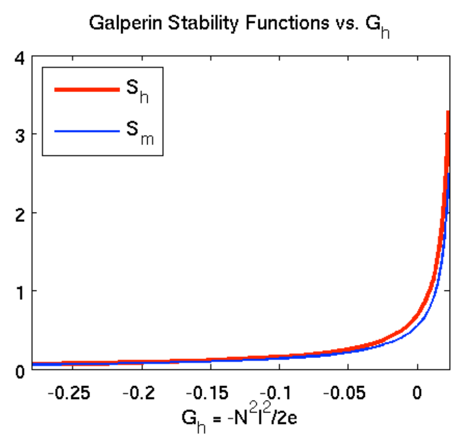

Fig. 8.2

To determine the evolution of q, we use the TKE equation

$$\frac{\partial}{\partial t}\left(\frac{q^2}{2}\right) = S + B + T - \varepsilon. \quad (8.18)$$

We model the shear and buoyancy production terms using eddy diffusion to find the fluxes:

$$S = -\overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \overline{v'w'}\frac{\partial \overline{v}}{\partial z} = lqS_m|d\mathbf{u}/dz|^2 \quad (8.19)$$

$$B = \overline{w'b'} = -lq\, S_h N^2 \tag{8.20}$$

We model the transport term by neglecting pressure correlations and using eddy diffusion to model the flux of TKE:

$$\overline{w'p'} \approx 0,$$

$$\overline{w'e'} = -K_q \frac{\partial}{\partial z}\left(\frac{q^2}{2}\right) = -lqS_q \frac{\partial}{\partial z}\left(\frac{q^2}{2}\right) \text{ (S_q is often taken to be 0.2)}$$

$$T = -\frac{\partial}{\partial z}\left(\overline{w'e'} + \frac{1}{\rho_0}\overline{w'p'}\right) = -lqS_q \frac{\partial}{\partial z}\left(\frac{q^2}{2}\right) \tag{8.21}$$

Lastly, the dissipation term is modelled in terms of characteristic turbulent velocity and lengthscales. While the lengthscale in ε is related to the master lengthscale, it is necessary to introduce a scaling factor to get the TKE to have the right magnitude:

$$\varepsilon = q^3/(lB_1),\; B_1 \approx 15. \tag{8.22}$$

With these forms for all the terms in the TKE equation, it can now be integrated forward in time.

Fig. 8.3 compares θ and u computed using this approach with observations from two days of the Wangara experiment; overall the agreement is very good. Fig. 8.4 shows the predicted time-height evolution of TKE, showing the development of a deep layer of vigorous turbulence during the day and a shallow layer of weak turbulence at night.

The basic improvement in using TKE vs. 1st order closure is that it includes TKE 'transport' (parameterized as eddy diffusion) and storage. In slowly evolving stable boundary layers, TKE storage and transport are negligible compared to local shear and buoyancy production of TKE. In this case, we expect that the TKE scheme should somehow reduce to 1^{st}-order closure. This is indeed the case as we now show. Without storage and transport, the TKE equation reduces to a three-way balance between TKE dissipation and shear and buoyancy production:

$$\varepsilon \quad = \quad S \quad + \quad B$$

$$q^3/(lB_1) = lqS_m|d\mathbf{u}/dz|^2 - lqS_hN^2$$

$$q^2 = B_1 l^2\{ S_m|d\mathbf{u}/dz|^2 - S_hN^2\} = B_1 l^2\, |d\mathbf{u}/dz|^2\{S_m - S_h\mathrm{Ri}\}$$

Hence, q can be found in terms of the local shear and stratification. The stability functions depend on G_m, but this also can be related to Ri:

$$G_m = -\, l^2N^2/q^2 = -\mathrm{Ri}/\,(B_1\{S_m - S_h\mathrm{Ri}\})$$

Hence we recover the first order closure method with

$$K_a = lqS_a = l^2|d\mathbf{u}/dz|F_a(\mathrm{Ri}),\;\; F_a = B_1^{1/2}\{S_m - S_h\mathrm{Ri}\}^{1/2}S_a,\;\; a = m, h$$

Fig. 8.5 plots the Galperin stability functions vs. Ri and the corresponding 'Smagorinsky functions' $F_{m,h}$(Ri) for TKE closure and compares the latter with their Monin-Obukhov counterparts (8.14). The similarity is striking and reassuring. Thus, use of this TKE closure for stable boundary layers will closely mirror the results of traditional first-order closure.

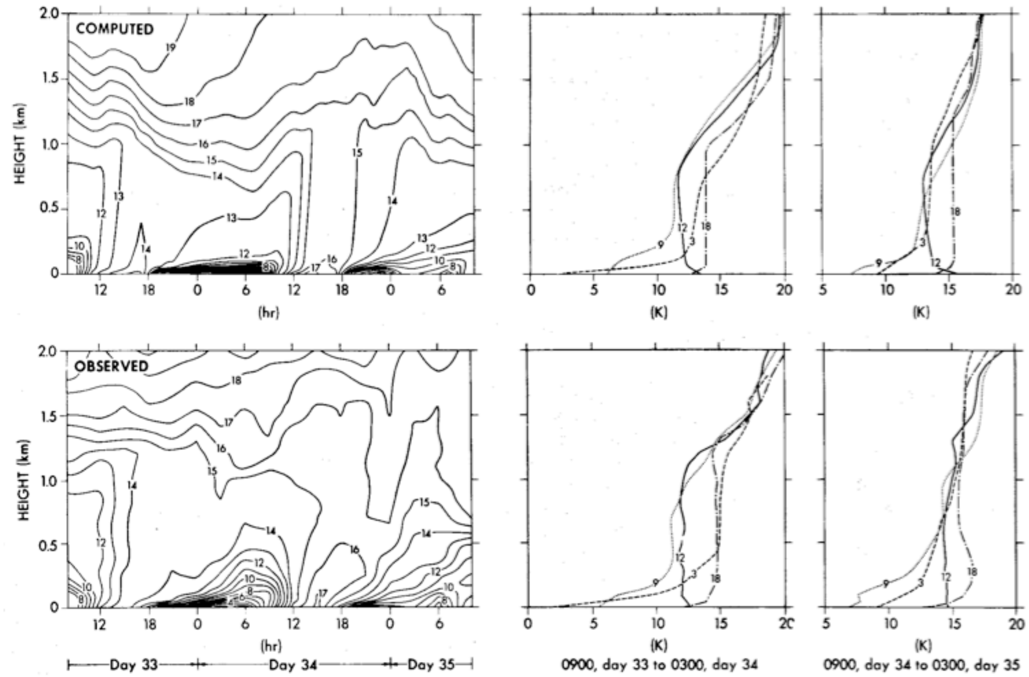

Fig. 11. Observed and calculated atmospheric boundary layer and vertical and temporal variations of mean virtual potential temperature −273°K. Units are degrees Kelvin.

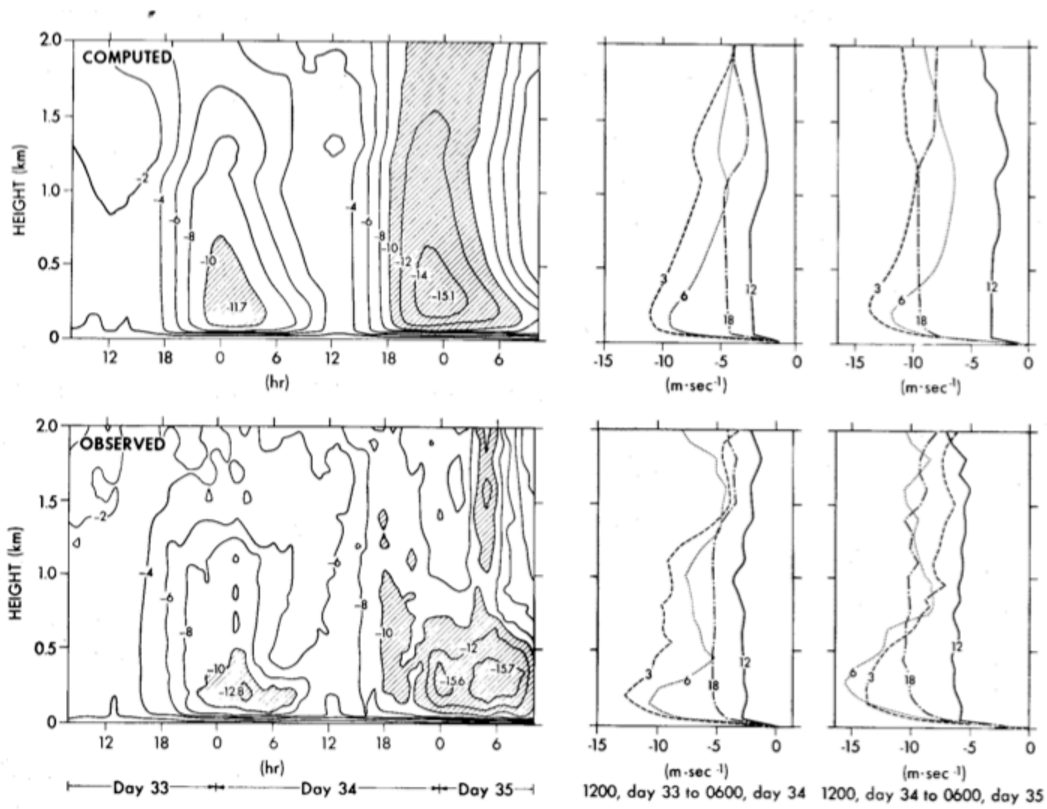

Fig. 12. Observed and calculated atmospheric boundary layer and vertical and temporal variations of the eastward mean wind component. Units are meters per second.

Fig. 8.3: Virtual temperature and *u* velocity simulated using 1.5-order closure and observed during two days of the Wangara experiment (Mellor and Yamada 1982)

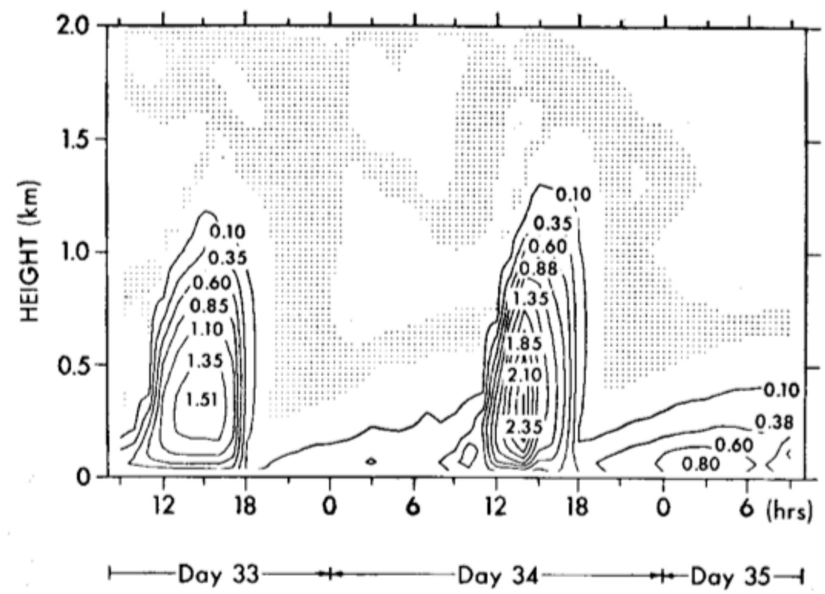

Fig. 14. Time and space variation of computed q^2 (twice the turbulent kinetic energy); units are square meters per second. The stippled areas indicate regions where $10^{-3} < q^2 < 10^{-2}$ m^2 s^{-1}.

Fig. 8.4

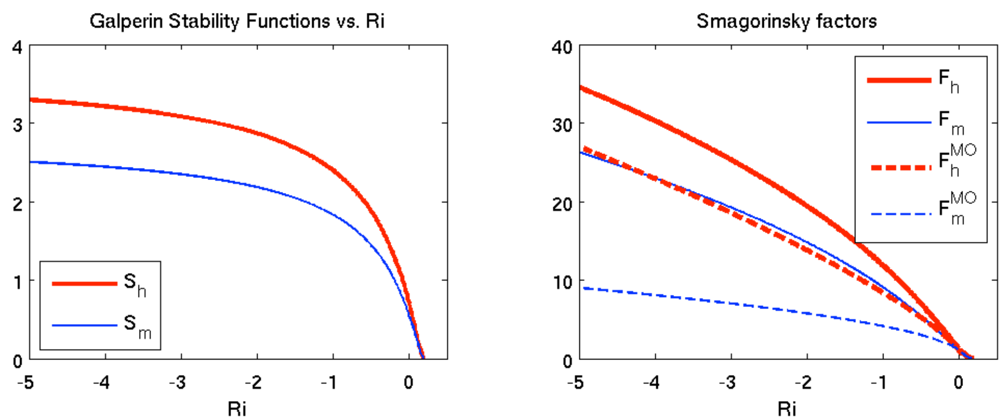

Fig. 8.5

K-profile methods

For specific types of boundary layer, one can use measurements and numerical experiments to specify a profile of eddy diffusivity which matches the observed fluxes and gradients. This can be particularly useful in situations such as stable nocturnal BLs which can be difficult for other methods. Such methods require a diagnosis of BL height h, then specify a profile of K. For instance, Brost and Wyngaard (1978) combined theoretical ideas and observational analysis to proposed the following profile for stable BLs:

$$K_m = ku_*hP(z)/(1 + 5z/L), \quad P(z) = (z/h)(1 - z/h)^{3/2}$$

This method is designed to approach the correct form $ku_*z/\phi_m(z/L)$ in the surface layer ($z/h \ll 1$). Similar approaches have been used for convective BLs. Advantages of the K-profile method are that it is computationally simple and works well even with a coarsely resolved BL, as long as the

BL height h can be diagnosed accurately. On the other hand, it is tuned to specific types of BL, and may work poorly if applied more generally than the situations for which it was tuned.

Comments on local closure schemes

First-order closure is most appropriate for neutral to weakly stable BLs in which little transport of TKE is occuring and the size of the most energetic eddies is a small fraction of the BL depth. In this case, it is reasonable to hope that the local TKE will be dependent on the local shear and stability, and that since the eddies are small, they can be well represented as a form of diffusion.

It also works tolerably well in convective boundary layers, except near the entrainment zone. Transport of TKE into the entrainment zone is usually required to sustain any turbulence there, because it is typically strongly stratified and buoyancy fluxes are therefore a local TKE sink. Since TKE transport is ignored in 1st order closure, there is no way for such a model to deepen by entrainment through an overlying stable inversion layer, as is observed. BL layer growth must instead be by encroachment, i. e. the incorporation of air from above the BL when its density is the same as within the BL. This does allow a surface-heated convective boundary layer to deepen in a not too unreasonable manner (Lecture 12), but creates severe problems for cloud-topped boundary layer modeling. Almost all large-scale models (e. g. CAM, ECMWF, and WRF) include a first-order closure scheme to handle turbulence that develops above the BL (due to Kelvin-Helmholtz instability or elevated convection, for instance).

1.5-order closure is also widely used, especially in mesoscale models where the timestep is short enough not to present numerical stability issues for the prognostic TKE equation. The Mellor-Yamada and Gayno-Seaman PBL schemes for WRF are 1.5 order schemes that include the effect of saturation on N^2. The Burk-Thompson scheme for WRF is a 1.5 order scheme with additional prognostic variables for scalar variances ('Mellor-Yamada Level 3'). The TKE equation in 1.5-order closure allows for some diffusive transport of TKE. This creates a more uniform diffusivity throughout the convective layer, and does permit some entrainment to occur. Quite realistic simulations of the observed diurnal variation of boundary layer temperature and winds have been obtained using this method (see figures on next page). However, getting realistic entrainment rates for clear and cloud-topped convective BLs with this approach requires considerable witchcraft. The BL top tends to get locked to a fixed grid level if there is a significant capping inversion and vertical grid spacing of more than 100 m or so. TKE closure has also proved successful for cloud-topped boundary layers, but only with grid spacings at the lower limite of what is currently feasible for GCMs. Grenier and Bretherton (2001, *MWR*, **129**, 357-377) showed that 1.5-order closure works well for convective BLs even at coarse resolution when combined with an explicit entrainment parameterization at the BL top, implemented as an effective diffusivity $K_e = w_e \Delta z$, where w_e is the entrainment rate, parameterized in terms of TKE, turbulent lengthscale, inversion density jump, etc., and Δz is the grid spacing at the BL top. This approach is used in the UW moist turbulence scheme in CAM5 (Bretherton and Park 2009)

K-profile methods are widely used in GCM BL parameterizations (e.g. the Holtslag-Boville scheme in CAM4). For convective boundary layers, a nonlocal contribution is often also added to the scalar fluxes to account for the effects of organized eddies filling the entire boundary layer (see Lecture 9). The nonlocal contribution can improve the predicted vertical profile of advected scalars within the BL.