Homework 5

Due Wed. 1 June, 2005

Fourier pseudospectral simulation of vortex dynamics.

Consider incompressible inviscid fluid flow in a periodic domain $0 < x, y < 1$. The flow is described by a streamfunction $\psi(x, y, t)$ such that $(u, v) = (-\psi_y, \psi_x)$. The initial streamfunction is an elongated isolated vortex

$$\psi(x, y, 0) = -0.25 \exp([-4(x - 0.5)^2 - (y - 0.5)^2]/2\sigma^2)$$

where $\sigma = 0.15$. If we define the vorticity $\zeta(x, y, t) = v_x - u_y \; = \psi_{xx} + \psi_{yy}$, the flow evolves according to the vorticity equation

$$D\zeta/Dt = \zeta_t - \psi_y \zeta_x + \psi_x \zeta_y = 0.$$

A Fourier pseudospectral method is ideal for accurately solving this equation.

Implement a pseudospectral method with RK4 time differencing to solve the vorticity equation. Use a 2D DFT in x and y (`fft2` in Matlab) with $N = 64$ Fourier modes in each direction, and a timestep $\Delta t = 0.25\Delta x$, where the grid spacing $\Delta x = 1/N$. Note that any time you calculate an updated $\zeta(x, y, t)$ you must also solve a Laplace equation with appropriate BCs to derive the corresponding $\psi(x, y, t)$. Also note that you will need to use a 2D DFT to numerically differentiate $\psi(x, y, 0)$ to calculate the initial $\zeta(x, y, 0)$. Contour-plot $\psi(x, y, t)$ and $\zeta\psi(x, y, t)$ every 0.125 time units starting at $t = 0$ out to $t = 0.5$. Describe what happens to ζ.