Amath-Math 586/Atm S 581 Spring 2005

Homework 4 Solutions

1) The suggested nondimensional rescaling of the traffic flow equation gives:

$$0 = Q_T + C(Q)Q_X = (C/A)q_t + (U_0 B/A)(1-2q/AQ_0)q_x,\ \ 0 < x < BL.$$

The desired form can be obtained by setting $A = 1/Q_0$, $B = 1/L$, and $C = U_0B = U_0/L$.

2) The characteristic form of the nondimensional traffic flow equation is:

$$\frac{dq}{dt} = 0 \quad \text{on} \quad \frac{dx}{dt} = c(q) = 1-2q\,. \qquad (*)$$

Each characteristic Γ_τ can be labelled by the time τ at which it intersects $x = 0$. Then (*) implies

$$q \text{ is constant along } \Gamma_\tau: \quad x = (1-2q)(t-\tau)\ . \qquad (*')$$

If $\tau > 0$, the left boundary condition tells us the value $q(\tau) = 0.12\sin^2 \pi\tau$ along Γ_τ. If $\tau < 0$, the initial condition implies $q(\tau) = 0$. Treating τ as a parameter which varies from -1 to ∞, this together with (*')determines the solution for all $0 \le x \le 1$, $t \ge 0$ and in particular at $x = 1$. This is implemented and plotted as part of the Matlab script hw4p234.m (Fig. 1).

3) The Asselin-filtered leapfrog/centered-in-space method is implemented in hw4p234.m. The method qualitatively tracks the exact solution, but a spurious overshoot is visible for t slightly larger than 2 and an erroneous ‘shoulder’ appears for t near 2.4. Its max-norm error is 0.0591.

4) The Lax-Wendroff (LW) and MC methods are also implemented with the specified Δx and Δt in hw4p234.m The LW method also produces a spurious overshoot, but smaller than the leapfrog method, and has a max-norm error of 0.0445. The MC method, as expected, removes the undershoot and has the smallest max-norm error of 0.0258.

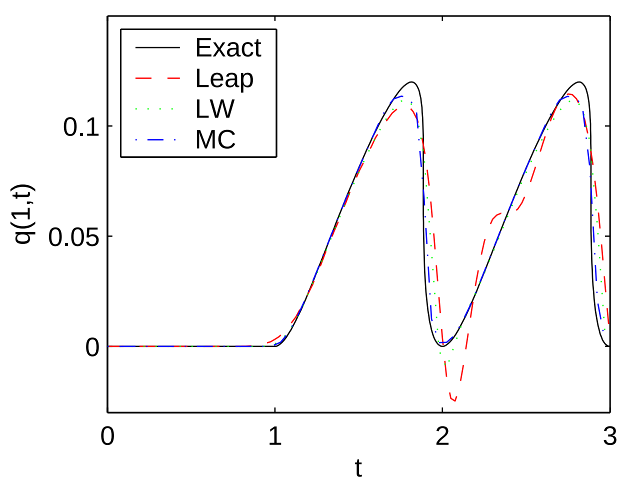

Fig. 1. The exact and finite volume solutions to the traffic flow problem at the right boundary.